# FRETTING STRESSES IN THE CONTACT OF COATED BODIES

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**Abstract:** *Complicated contact processes such as the fretting contact of dissimilarly elastic materials require very efficient numerical tools to overcome the complex algorithm structure, involving multiple nested loops, and the time consuming convolution products calculation. This paper presents a contact simulation technique derived from the Boundary Element Method, whose efficiency stems from the use of the Conjugate Gradient method and the fast Fourier transform. The behavior of the fretting contact of coated elastic materials in the first loops is simulated with an algorithm based on three nested loops, until a periodic stability is reached. The fine spatial and temporal resolution promises well-converged numerical solutions. The elastic response of the layered material is derived using closed form results calculated in the frequency domain. The method allows for the prediction of the subsurface stresses in the fretting contact. Subsequent identification of the intensity and position of the maximum equivalent stress provides insight on the propensity of plastic yielding in the coated material.*

**Keywords:** *numerical simulation, fretting contact, coating, stress state*

## 1. Introduction

Mechanical contacts under load are subject to vibrations that induce repeated relative surface motions of small amplitude (as low as 3 nanometers), also known as fretting. This condition induces damages, i.e. wear and fatigue, which decrease the service life of the contacting element. The use of protective coatings is proven to provide lower friction, higher wear resistance and longer fatigue life.

Prediction of contact processes in fretting conditions requires complex mathematical models without analytical solution, whose numerical solution is very computationally intensive with standard methods such as the finite element analysis.

A special technique derived from the Boundary Element Method was developed for Contact Mechanics, integrating modern numerical tools such as the Conjugate Gradient method and the fast Fourier transform. This method allowed for the solution [1-5] of the fretting contact of homogeneous materials, by taking into consideration the interdependence between the normal and the shear contact tractions.

Important research efforts [6-10] were directed toward the study of the behavior of layered materials, concluding [11] with the calculation of the response to unit point loads in the frequency domain. These closed-form solutions can be further used [12] in conjunction with the convolution theorem to calculate the layered medium response to prescribed surface loading.

This paper explores an algorithm for the fretting contact of dissimilarly elastic materials and presents new results concerning the stress state in the fretting contact. State-of-the-art numerical tools allow for well-converged solutions.

## 2. Algorithm outline

The fretting contact model relies on the formulation of the Cattaneo-Mindlin problem [13,14], which proves that the hypotheses of a fully sticking contact area in a frictional contact under normal and tangential load leads to infinite shear tractions at the boundaries of the contact region. Slip is introduced to relieve the latter stresses that are not compatible with the Linear Theory of Elasticity, and the contact state is referred to as "partial slip", i.e. the contacting bodies are globally sticking, but there exist micro-slip at the edges of the contact regions. This contact framework, coupled with a loading history in which the normal load is kept constant whereas a tangential force oscillates around zero but has a limited magnitude so that full slip does not occur, describes best the fretting contact process.

A schematic of the contact model is depicted in figure 1, in which the equivalent punch is a spherical homogenous indenter whereas the equivalent half-space is a coated material whose substrate elastic properties, i.e. the Young modulus $E_s$ and the Poisson's ratio $\nu_c$ match those of the indenter. The equations describing the aforementioned model are well developed [15] in the literature of the Contact Mechanics. Model digitization lead to a linear system of equations having the discrete contact tractions as unknowns. The system resolution is best achieved by splitting the contact model in two subsystems, one having the contact pressure and the other one the shear tractions as unknowns.

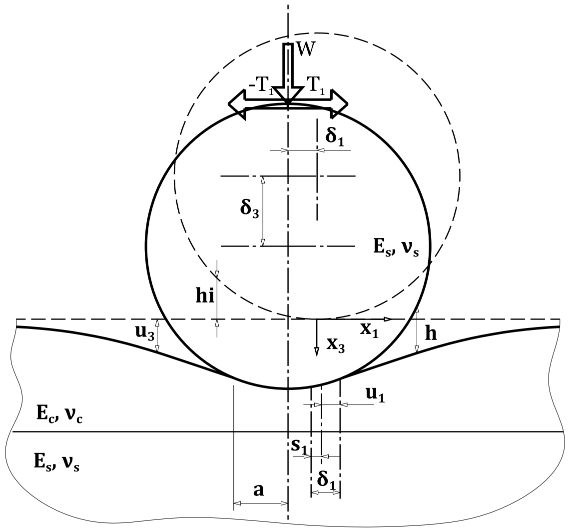


**Figure 1:** Fretting contact schematic

The two subsystems are independent only in the case of similarly elastic materials. The flow-chart of the algorithm for the solution of these subsystems, which is based on the solution of the frictionless rough normal contact advanced by Polonsky and Keer [16], is depicted in figure 2. The Conjugate Gradient method is engaged together with the DCFFT technique [17] for rapid computation of convolution products in the frequency domain. The latter algorithm was adapted by Spinu and Glovnea [4] to obtain the solution of the Cattaneo-Mindlin problem. Whereas in the subsystem for the normal contact direction, the contact area has to be isolated from the computational domain and the contact pressure has to be calculated, in the subsystem for the tangential direction the partition of the contact area in the slip and the stick regions has to be identified, as well as the shear tractions.

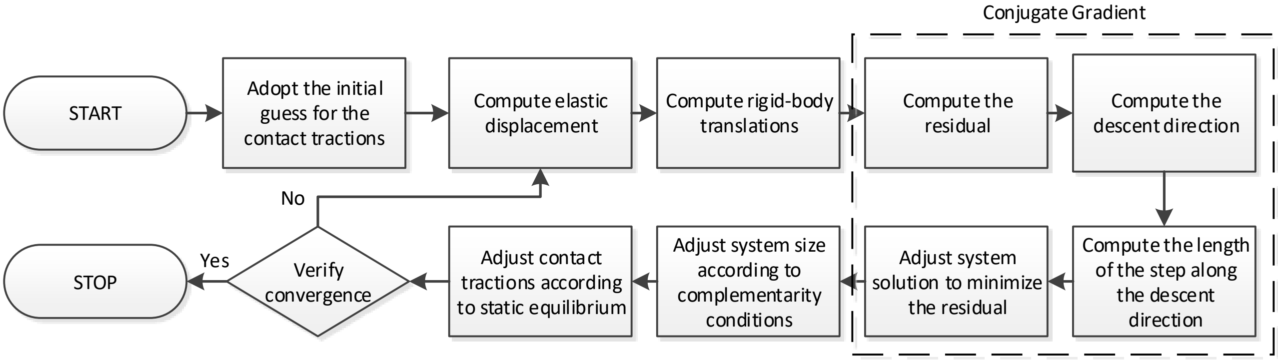


**Figure 2:** Algorithm flow-chart for the frictionless rough normal contact

The algorithm presented in figure 2 can solve the systems resulting from either of the following equations:

$$C_{31} \otimes q_1 + C_{32} \otimes q_2 + C_{33} \otimes p = h - hi + \delta_3, \quad (1)$$

$$\begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \end{bmatrix} \otimes \begin{bmatrix} q_1 \\ q_2 \\ p \end{bmatrix} = \begin{bmatrix} s_1 + \delta_1 \\ s_2 + \delta_2 \end{bmatrix}, \quad (2)$$

but only $p$ is considered as unknown in equations (1), whereas only $q_1$ and $q_2$ are unknown in equation (2). The solution of the Cattaneo-Mindlin problem involves the resolution of equation (1) with $C_{31} \equiv C_{32} \equiv 0$, followed by the resolution of equation (2) with $C_{13} \equiv C_{23} \equiv 0$.

When the properties of the elastic materials are dissimilar, as in the contact process depicted in figure 1, all influence coefficients $C_{ij}$, $i, j = 1,2,3$, have non-vanishing values, and therefore an iterative strategy has to be considered, as depicted in figure 3. Equations (1) and (2) are solved successively with guess values for the contact tractions until convergence is reached. The flow-charts presented in figures 2 and 3 form a two nested loops strategy that solves the instantaneous contact state.

A supplementary outer loop assures that the loading path is accurately reproduced, as requested for dissipative processes such as the frictional slip. The load should be applied in small increments, as shown in figure 4.

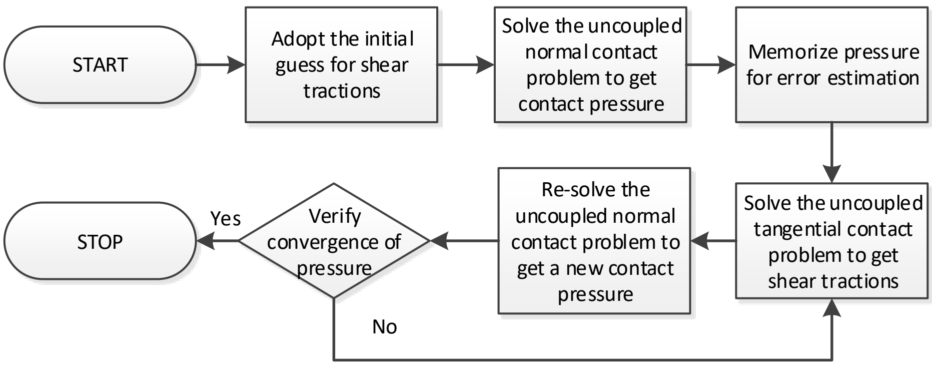


**Figure 3:** Algorithm flow-chart for the contact of dissimilarly elastic materials

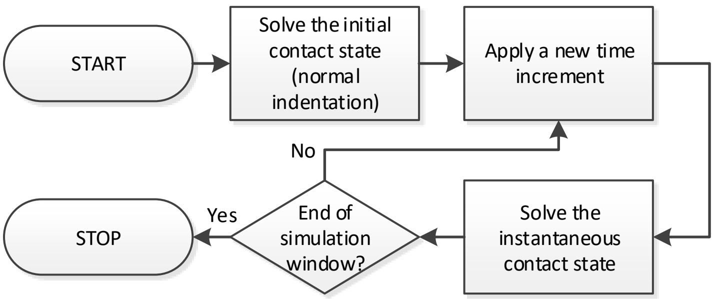


**Figure 4:** Reproduction of the loading path

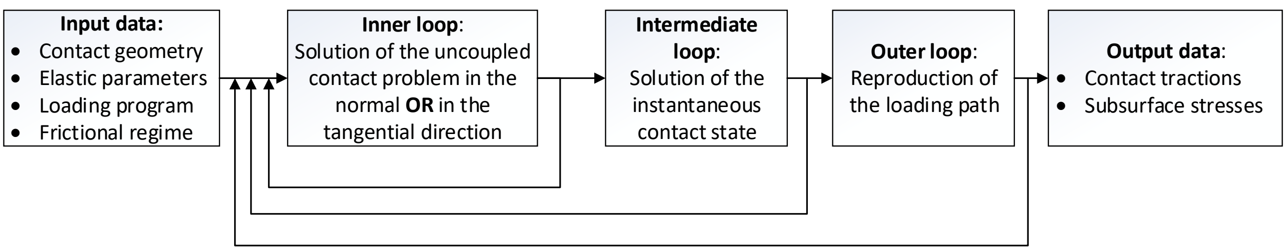


**Figure 5:** The loops in the algorithm for the fretting contact

The resulting three-loop algorithm, aggregated in figure 5, provides the contact tractions that can subsequently be used to compute subsurface stresses. The latter calculation is performed according to the method indicated in [12], by making use use of the closed form expressions of the frequency response functions for stresses in layered materials. The convolution product is calculated in the frequency domain, as element-wise multiplication between the discretized frequency response function and the transform of the contact traction. The latter result is then transferred in the space domain by inverse fast Fourier transform:

$$\sigma_{ij} = IFFT(\hat{f} \cdot \hat{q}_i)\,. \qquad (3)$$

## 3. Results and discussions

The fretting contact is simulated with the algorithm described in the previous section, for the loading history depicted in figure 6. The simulation window is large enough for the contact to reach a periodic stability. The stress state is calculated for the points A, B and C from the loading path. The iso-contours of the second invariant of the stress tensor, normalized by the Hertz pressure corresponding to the case $E_c = E_s$ and $\nu_c = \nu_s$, are depicted in figures 7-9 for various elastic moduli ratios between the coating and the substrate, whereas $h_c/a_H = 1$.

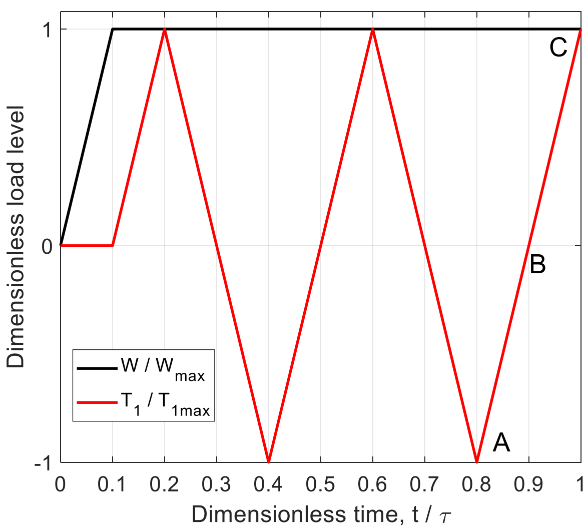


**Figure 6:** The loading history

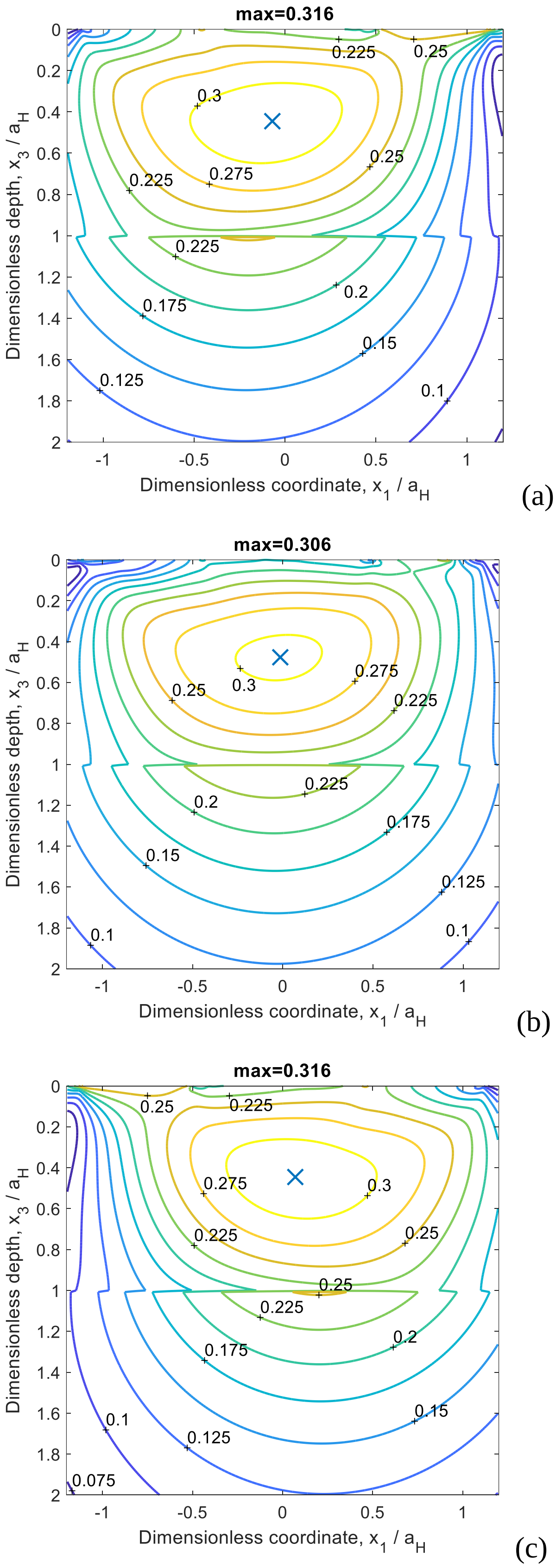


**Figure 7:** Iso-contours of $J_2/p_H$ in the plane $x_2 = 0$, $E_c/E_s = 0.5$ : a) in A; b) in B; c) in C

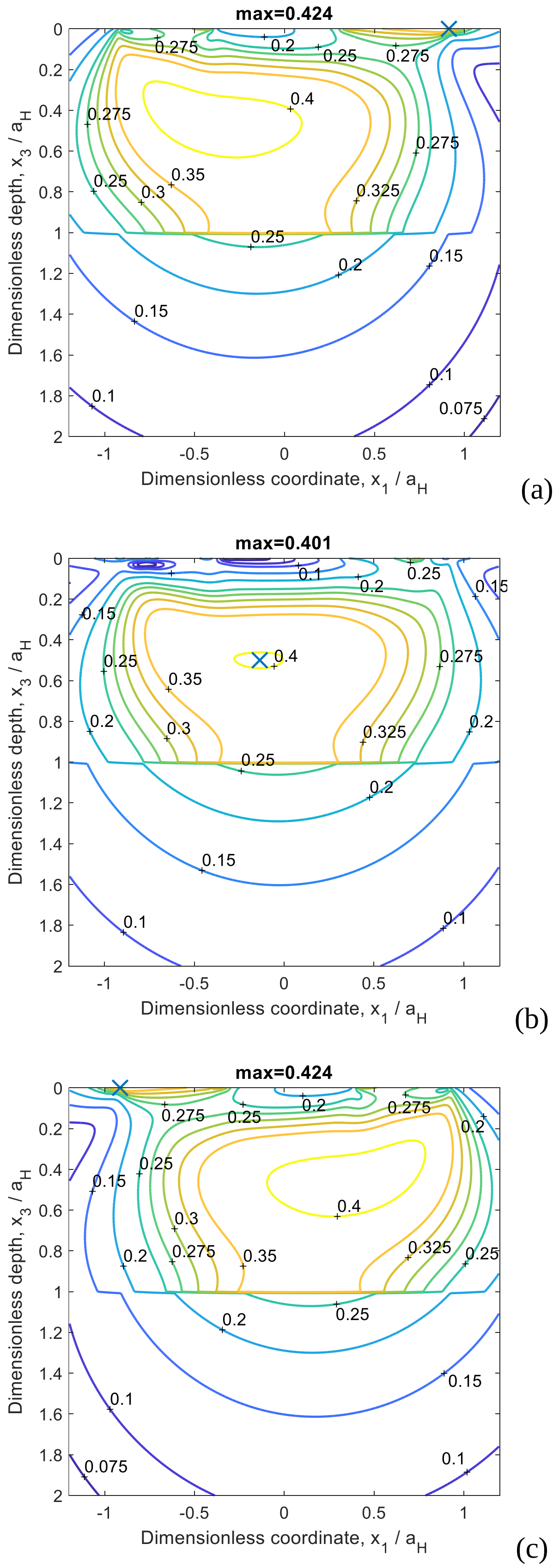


**Figure 8:** Iso-contours of $J_2/p_H$ in the plane $x_2 = 0$, $E_c/E_s = 2$: a) in A; b) in B; c) in C

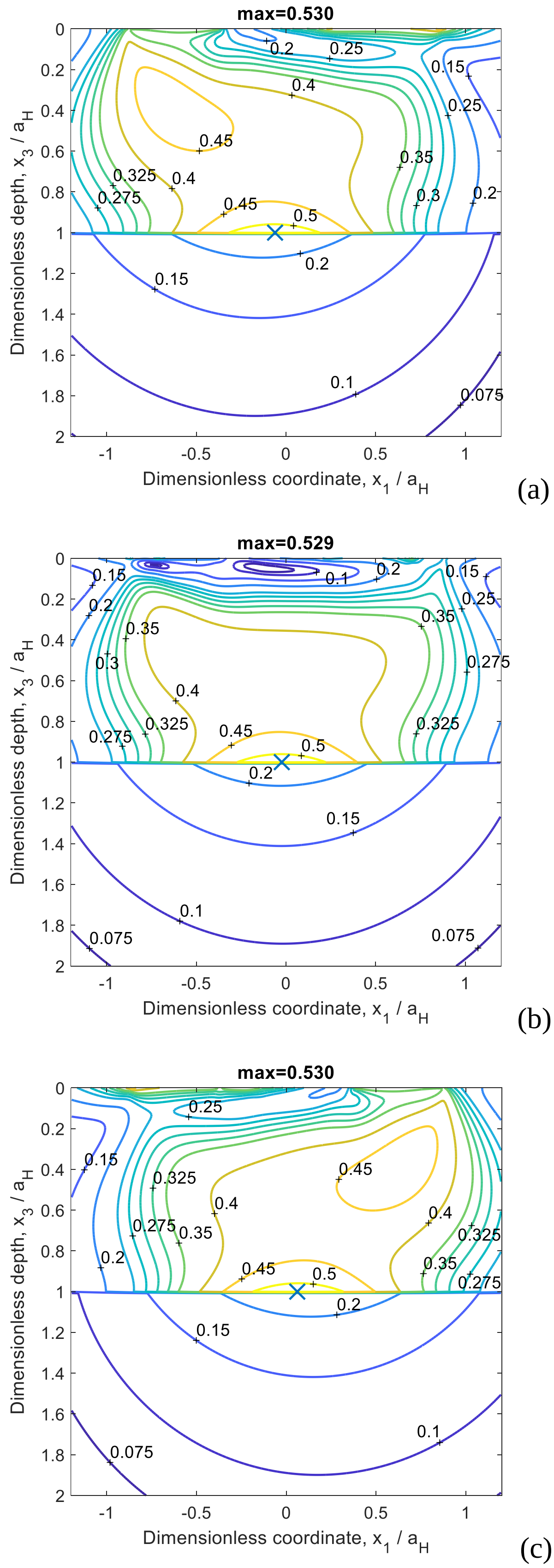


**Figure 9:** Iso-contours of $J_2/p_H$ in the plane $x_2 = 0$, $E_c/E_s = 4$: a) in A; b) in B; c) in C

## 4. Nomenclature

$E_c, E_s$ - Young moduli of the coating and substrate, respectively
$\nu_c, \nu_s$ - Poisson's ratios of the coating and substrate, respectively
$W, T$ - normal and tangential forces
$q_i, i = 1, 2$ - shear tractions
$p$ - contact pressure
$a$ - contact radius
$a_H, p_H$ - Hertz contact parameters
$h, hi$ - gap, initial gap
$h_c$ - coating thickness
$u_i, i = 1, 2, 3$ - displacement
$\delta_i, i = 1, 2, 3$ - rigid-body translations
$s_i, i = 1, 2$ - slip distances
$C_{ij}, i, j = 1, 2, 3$ - influence coefficients
$f$ - frequency response function
$\mu$ - frictional coefficient
$\sigma_{ij}, i, j = 1, 2, 3$ - subsurface stresses
$J_2$ - second stress tensor invariant
$\tau$ - length of the simulated window
$\hat{}$ - discrete samples in the frequency domain
$\otimes$ - convolution product

## 5. Conclusions

The behavior of the fretting contact of dissimilarly elastic materials is simulated using a numerical technique originally developed for homogeneous materials. The elastic response of the layered medium is assessed based on the existing frequency response functions.

The use of state-of-the-art numerical tools such as the Conjugate Gradient method for the solution of linear systems and the fast Fourier transform allows for well-converged numerical solutions.

The fretting contact is simulated until a periodic stability is reached, and then the stress is calculated for extreme values of the tangential force. The influence of the coating thickness on the intensity and position of the maximum equivalent stress is assessed.

## 6. Acknowledgement

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