EXPERIMENTAL STUDY OF RUBBER SPHERES IN CONTACT

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Abstract: The aim of this paper is to presents some aspects regarding rubber sphere's behavior under loading. In order to achieve this goal, an experimental device was conceived and used. The experimental results were used to compare four cases of rubber spheres assemblies and to obtain the principal factor which influences the contact deformation of the package. Also, the experimental results were used to see if the shape of the load – deformation curve obtained for nonlinear elastic materials is similar to the one obtained using hertz theory equations.

Keywords: *rubber, circular contact, nonlinear elastic, contact deformation*

1. Introduction

In modern mechanical systems manufacturing, rubber is frequently used due to its mechanical elasticity properties and durability. Nowadays, several rubber types are known, such materials being used in many domains with properties adapted to the practical application requirements. As there are many types of rubbers, with mechanical properties which cover a wide range of necessities, the rubber products can be used in almost all areas such as: metallurgy and siderurgy, chemical plants and mining holdings, agriculture and food industry, building and woodwork, shipments and car services, shipyards, gyms and runways, pharmacies and the list could continue endlessly, [1].

Rubber's working capacity depends on its main properties as: elasticity, fatigue strength as well as hysteresis losses due to variable loadings. Rubber's capacity to support large stretches distinguishes it from other materials. The rubber specimens can be stretched up close to rupture without suffering large residual deformations. Loading curves have large differences between first stretch and further ones especially for filled specimens. After a few loading-unloading cycles, the shape of the stress-strain correlation curve stabilizes (the variation of the stretch curve at the first cycles is due to irreversible variation of rubber's structure which appears in these cycles). The evolution of the characteristic stress-strain curves for consecutive loading-unloading cycles is represented graphically in Figure 1. At the first loading of the rubber specimen, the hysteresis loop is very large and sometimes can achieve 50% from the area contained under the characteristic stress-strain curve. For subsequent loads, the hysteresis loop is shrinking reaching a minimal value when the elastic properties completely stabilized. [Po63].

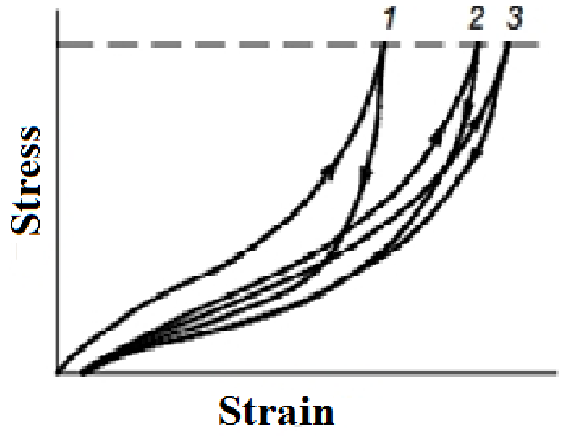

Figure 1. *Stress-strain curves for successive loading – unloading cycles, [Po 63]*

Also, from Figure 1 can be noticed that rubber's stress – strain correlation curve has a nonlinear shape. The mechanical contacts between nonlinear elastic bodies will have different behavior and they will have other stress and strain states from those obtained in the same condition for the contact between linear elastic bodies. Due to the use in such large areas and also the special behavior to other materials, a thorough research of rubber's contacts is imposed.

2. Aspects regarding the point contact between linear and nonlinear bodies

The contact mechanics study the stress and strain states from the bodies in one or several contact points. Mathematically and physically the contact problem can be expressed using continuum medium mechanics and also elasticity and plasticity theory.

The mechanical contacts have a large variety of body shapes, dimensions and loads, therefore, in view of a systemic approach, it is necessary to classify them. There are several criteria such as: the contact configuration for the particular case when the loading tends to zero, stress and strain states from the contact bodies, the correlation between the relative motion of the surface points and the variation of the normal loading, lubrication regime, the friction between surfaces and the considered boundary surfaces type, etc. [Di 09]. Regarding first criterion, the contacts can be concentrated also named nonconforming contacts and on surface, named conforming contacts. Conformity consist in the tendency of covering between one surface to the other one while total conformity consist in a complete superposition between contact surfaces, points by point, on a certain area.

Nonconforming contacts (or concentrated) appear for the case when the load tends to zero, and those two contact surfaces have points placed on geometric spot with null area. This geometric spot can be resumed to a point, contact named point contact or a curve, contact named linear contact. Regarding geometrical particularities of the boundary surfaces in the initial contact region, the contacts can be hertz and non-hertz. The concentrated contacts are hertz if on the initial contact point or curve, as well as near the region where the surfaces is going to be in contact, those two boundary surfaces doesn't have singular points, meaning that each point from the considered region has a single normal line and a single tangent plane, and the bodies materials are linear elastic. Different geometric conditions imposed to the surfaces or materials properties leads to the occurrence of the non-hertz contacts.

The contact between spheres, which at limit can be a sphere-plane contact, it is a circular contact. In this case, the contact bodies curvatures became equal in principal planes and are the same for both bodies, $\boldsymbol{k_{1x}} = \boldsymbol{k_{1z}}$ and $\boldsymbol{k_{2x}} = \boldsymbol{k_{2z}}$, meaning $\boldsymbol{k_x} = \boldsymbol{k_z}$. By applying a normal loading, this type of contact expands around the initial contact point a circular area of a radius. Also, circular area appears in the contact between circular cylinders with perpendicular axis.

Circular contact parameters have the following expressions, [Jo85]:

$$a = \sqrt[3]{\frac{3}{2}\frac{\eta Q}{k}}, \tag{1}$$

$$\delta = \frac{1}{2}\sqrt[3]{\frac{9}{4}\eta^2 k Q}, \tag{2}$$

$$p_o = \frac{1}{\pi}\sqrt[3]{\frac{3}{2}\frac{k^2}{\eta^2}Q}, \tag{3}$$

$$\eta = \frac{1-\nu_1^2}{E_1} + \frac{1-\nu_2^2}{E_2}, \tag{4}$$

$$k = \frac{1}{R_{1x}} + \frac{1}{R_{1z}} + \frac{1}{R_{2x}} + \frac{1}{R_{2z}} \tag{5}$$

x - coordinate in tangent plane;
where: a – contact area radius;
δ – normal approach between contact bodies;
p_0 – maximum pressure in contact;
η – contact stiffness;
Q- normal loading;
k- equivalent contact curvature;
E_1, E_2- elasticity modulus
ν – transverse coefficient (Poisson ratio)
R – curvature radius

Contacts between nonlinear elastic bodies have stress and strain states different from those obtained for the linear elastic bodies in contact. For studying these types of contacts, it is necessary to know the contact bodies' behavior due to their respective material properties.

Generally, in modern practical applications many materials with nonlinear elastic properties are used. The elastic behavior of the materials can be interpreted as a particular case of nonlinear elasticity. Usually, the materials have linear elastic behavior only in some domains, having linear elastic characteristics in the initial phase and afterwards they become nonlinear. [Fa 04].

3. The experimental setup

In order to study the correlation between loads and normal approach of rubber spheres and also for highlighting the deviation between the rubber's contact and the hertz contact an experiment was proposed that allows tracing the load-normal approach correlation for two and three spheres packages under the effect of a load along the centers axis.

Some difficulties appear in experiments. The first technical problem that hat to be surpassed was the applying of loads along the axis which passes through contact bodies centers. For this purpose, a guiding device of the contact spheres was conceived. The spheres were placed in guiding supports, cylindrical cavities with flat or spherical ends.

The relative positioning of the spheres and their displacement was possible by using two cylindrical columns fixed by one of the supports with cylindrical cavities and mounted with tight tolerances from the others, Figure 2.

In order to make a study in which the deformation of the spheres to be separable, a compression test was made in two versions, with empty cylindrical supports and a second test with the supports filled with sculpture plaster applied such as to fully fill the empty volume between spheres and the interior walls of the support.

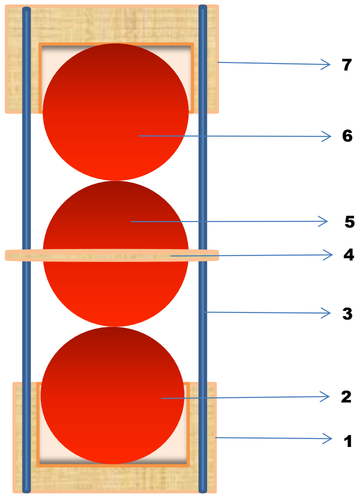

Figure 2. *Guiding and loading device*

where, 1 – lower support
2 – lower sphere
3 – guiding columns
4 – guiding ring
5 – middle sphere
6 – upper sphere
7 – upper support

Two devices were manufactured corresponding to two sets of sphere dimensions. Tests were conducted with two and three contact spheres and with cylindrical or spherical supports. Thus, for the proposed experimental research regarding the rubber contact behavior, the following were used:

- Two sets of two and three identical spheres made from rubber having diameters of 42 mm and 56 mm, respectively.
- Guiding and supporting elements of the elastic bodies (Figure .4).
- Validator PLUS TM testing machine and its software.

For testing, the experimental device was mounted on the testing machine. The experimental testing procedure is managed by software which allows recording and display in real time of the experimental values. Also, the software can save and export the data, show maximum and minimum values, make statistics and charts in Word, Excel, PDF and other file types.

The testing methodology consists of mounting the experimental device between the testing machine plates, imposing a maximum relative approach to the spheres ensemble and the tester will provide the load – displacement correlation up to the imposed displacement value.

4. Experimental results

Several tests were made and the correlation between loads and displacements were highlighted. Four cases were investigated and the results are presented below:

1. The contact between 2 identical spheres compressed between supports with flat end cavities

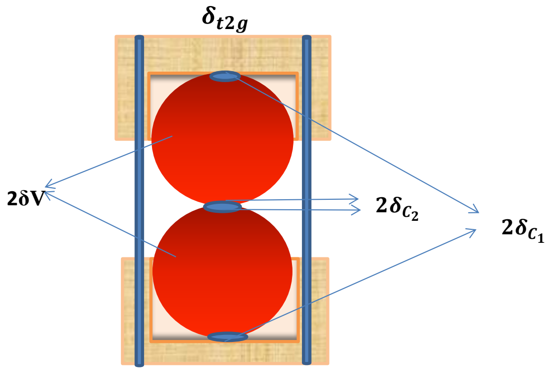

Figure 3. *The deformation of two rubber spheres compressed between supports with flat end cavities*

In this case, the loading causes to the spheres ensemble two contact deformations δ_{c_1} and two contact deformations, δ_{c_2}. The sum of those deformations represents the total deformation of the spheres ensemble.

$$2\delta_{c_1} + 2\delta_{c_2} = \delta_{t2g}, \qquad (6)$$

where:

-δ_{c_1} is the deformation of a rubber sphere pressed against a rigid flat surface;

-δ_{c_2} is the deformation of a rubber sphere pressed against another sphere;

-δ_{t2g} is the total deformation of two rubber spheres compressed between cavities with flat ends;

The load - deformation correlation was displayed, saved and printed using the testing machine's software. An example of the recorded data is presented in Figure 4.

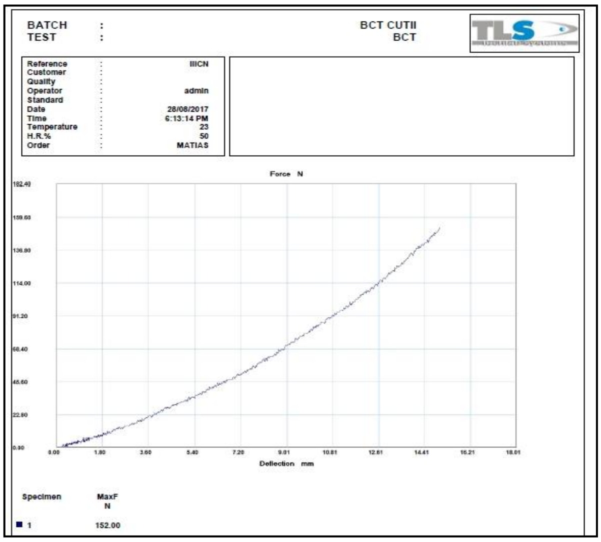

Figure 4. *Example of load - deformation correlation curve obtained on Validator PLUS TM for spheres compressed between empty cavities*

Several experimental values were extracted from the above curve and represented as shown in Figure 5.

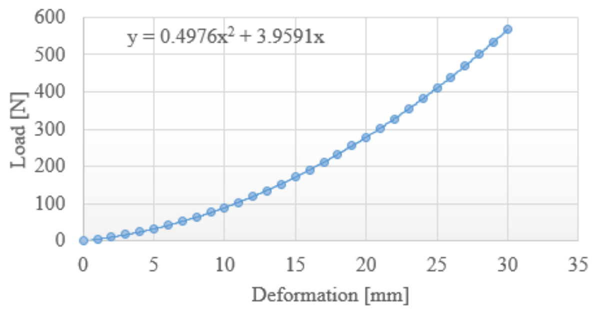

Figure 5. *Load –deformation correlation of two rubber spheres compressed between supports with flat end cavities*

The mathematical equation corresponding to the curve which interpolates the experimental values is:

$$y = 0{,}497x^2 + 3{,}959x \qquad (7)$$

2. The contact between 2 identical spheres compressed between spherical cavities

In this case the spheres fully fill the cavities from the supports. This fact makes that after applying load along the axis which passes through spheres centers, the total deformation supported by the rubber spheres to be in principal caused by the sphere-sphere contact, represented in Figure 6 and symbolized by δ_{c_2}, while the other deformation can be neglected.

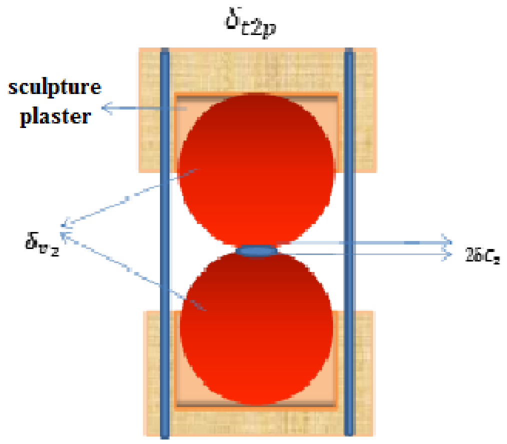

Figure 6. *The deformations of two rubber spheres compressed between supports with spherical cavities*

The total deformation of the ensemble will be:

$$\delta_{t2p} = 2\delta_{C_2}, \qquad (8)$$

where with δ_{t2p} symbolized the total deformation of the two rubber spheres compressed between supports with filled cavities.

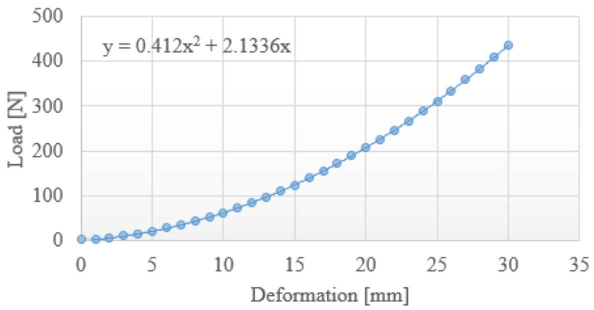

Figure 7. *Load - deformation correlation of two rubber spheres compressed between supports with spherical cavities*

Several values were extracted from the experimental data and those values were interpolated by a parabolic equation. The following form of the interpolating equation was obtained:

$$y = 0{,}412x^2 + 2{,}133x \qquad (9)$$

3. The contact between 3 identical spheres compressed between cylindrical cavities with flat ends

The spheres were compressed between supports with flat end cavities similar to the first presented case. To obtain valid results, the contact must be in a configuration which will assure that all three spheres will be arranged along the axis that passes through sphere centers and on the direction of loading. For this purpose, beside the two guiding columns, an additional ring was used, made from the same material as the end supports. The ensemble is represented in Figure 8.

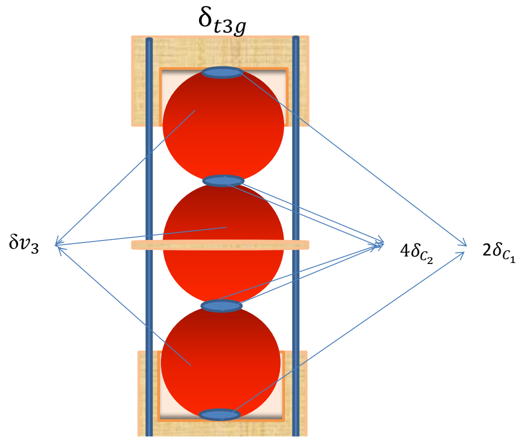

Figure 8. *Three rubber spheres compressed between supports with flat ended cavities*

In this case, the total deformation will be:

$$2\delta_{C_1} + 4\delta_{C_2} = \delta_{t3g} \qquad (10)$$

where - δ_{t3g} is the total contact deformation of three spheres compressed between supports with empty cavities;

***4.** The contact between 3 identical spheres compressed between supports with spherical cavities*

In this case the upper and the lower rubber spheres will be fixed in supports cavities filled with sculpture plaster. The equation of the ensemble deformation in this case will have the following form:

$$4\delta_{C_2} = \delta_{t3p}, \qquad (11)$$

where δ_{t3p} is the total axial deformation of the ensemble.

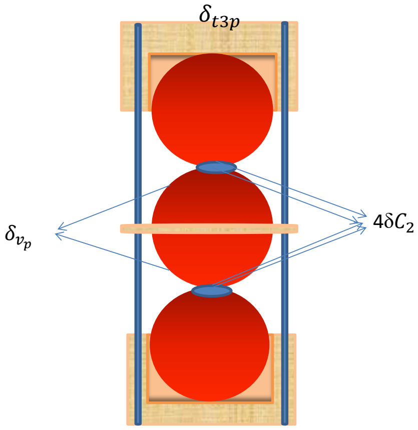

Figure 9. *The contact between 3 identical spheres compressed between supports with spherical cavities*

Starting from the equations of Hertz circular contact and particularization those for the presented cases, the following relations were obtained:

a. For the contact between two identical spheres the equivalent contact curvature will have the following form:

$$k = \frac{1}{R} + \frac{1}{R} + \frac{1}{R} + \frac{1}{R} = \frac{4}{R}, \qquad (12)$$

and the contact deformation symbolized with δ_{c_2} will become:

$$\delta_{c_2} = \frac{1}{2}\sqrt[3]{\frac{9}{4}\eta^2 kQ} = \sqrt[3]{\frac{4}{R}} \cdot \frac{1}{2}\sqrt[3]{\frac{9}{4}\eta^2 Q}, \qquad (13)$$

b. If the contact is between spheres - rigid flat plane the equivalent bodies' curvature and contact stiffness will become:

$$k_1 = \frac{1}{R} + \frac{1}{R} = \frac{k}{2}, \eta_1 = \frac{\eta}{2} \qquad (14)$$

and the contact deformation, symbolized in this case with δ_{c_1} will have the following form:

$$\delta_{c_1} = \frac{1}{2}\sqrt[3]{\frac{9}{4}\eta_1{}^2 k_1 Q} = \frac{1}{2}\sqrt[3]{\frac{9}{4}\frac{\eta}{2}\frac{k}{2} Q} \qquad (15)$$

Making the ratio between those two relations of the contact deformation, the next equation is obtained:

$$\frac{\delta_{c_1}}{\delta_{c_2}} = \frac{1}{\sqrt[3]{2}} = 0{,}793 \qquad (16)$$

from equation (16) results:

$$\delta_{c_1} \approx 0.793\, \delta_{c_2} \qquad (17)$$

Using equation (6), (8) and (16) the following equation system is obtained:

$$\begin{cases} 2\delta_{C_1} + 2\delta_{C_2} = \delta_{t2g} \\ 2\delta_{C_2} = \delta_{t2p} \\ \delta_{C_1} = 2\,\delta_{C_2} \end{cases} = \qquad (18)$$

$$= \begin{cases} 2\delta_{C_1} = \delta_{t2g} - \delta_{t2p} \\ 2\delta_{C_2} = \delta_{t2p} \end{cases}$$

Solving the equation system the below relations can be written:

$$\delta_{C2} = \frac{\delta_{t2g}}{3{,}6} = \frac{0{,}509x^2 + 2{,}528x}{3{,}6} = \qquad (19)$$
$$= 0.1414x^2 + 0.7024x$$

$$\delta_{c_1} = 0{,}8\,\delta_{c_2} = \qquad (20)$$
$$= 0.0995x^2 + 0.8684x$$

$$\delta_{v_1} = 2(\delta_{c_1} + \delta_{c_2}) = \qquad (21)$$
$$= 0{,}4577x^2 + 3{,}9239x$$

$$\delta_2 = 2\delta_{c_1} = 0{,}199x^2 + 1{,}7367x \qquad (22)$$

Representing graphically the interpolation equation of the experimental values obtained for 56mm spheres contact and the same contact geometry equation (2), the following chart was obtained:

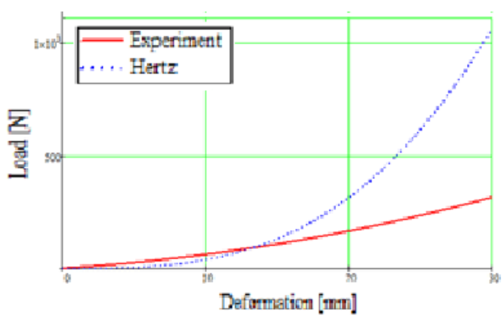

Figure 10. *Load - deformation correlation for 56 mm spheres contact obtained experimental and using Hertz relations*

5. Conclusions

By comparing the theoretical results presented above with those obtained experimentally, some conclusions can be highlighted:

The load - contact deformation correlations obtained from tests reveal that the dependency is nonlinear, confirming, once again, the theoretical results presented in literature.

The correlation curves obtained experimentally have similar evolutions to those presented by Diaconescu E. in 2009, at "Ştefan cel Mare " University of Suceava.

By interpolating the load – deformation correlation values with a polynomial function, it is possible to determine the value of load and deformation in any point.

From the second set of spheres, having 56mm in diameter, we notice that the deformation of rubber spheres is independent to loading history. This aspect can be observed from the below chart where the contact bodies were compressed for 10mm, 15mm and 30mm.

Can be seen that the curves approximates the same route (Figure 11). The material behavior is reversible when the material is subjected to cycling loads in adiabatic and isothermal conditions.

For low loads the computed values of deformations obtained with Hertz equations are bigger than those obtained experimental while at high loads deformations are smaller.

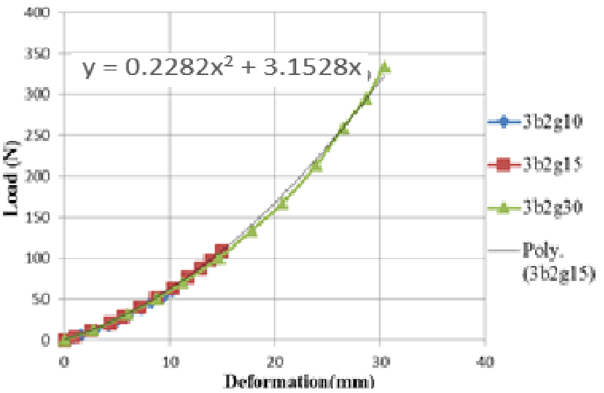

Figure 11. *Load – deformation correlation of two rubber spheres compressed gradually to 10mm, 15mm and 30mm displacement*

The deformation in contact for the case in which the spheres compressed between empty supports are bigger than those obtained for the case in which the spheres are compressed between filled supports. The total deformation of the ensemble is equal with the sum of two contact deformations.

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