ON DIRECT COUPLING OF TWO SHAFTS. PART 2: KINEMATICAL ANALYSIS

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Abstract: *The paper performs the kinematical analysis of a direct coupling solution between thwo shafts with crossed axes. Using the Hartenberg-Denavit convention for denoting the axes of the frames and additionally the matrix form of point contact condition, a system of equations is obtained. The solutions of this system are the variable parameters from the pairs of the mechanism, obtained as functions of the driving element motion. An actual example is presented in the last part of the paper.*

Keywords: *crossed shafts, kinematical analysis, Hartenberg-Denavit convention*

1. Problem formulation

Two kinematical elements 1 and 2 are considered, Fig.1, connected to the ground through revolute pairs A and B. The two elements creates a point contact in C. More specifically, the point C is the intersection between a branch of element *1* parallel to the axis of rotation and a branch of element *2* perpendicular to the axis of rotation.

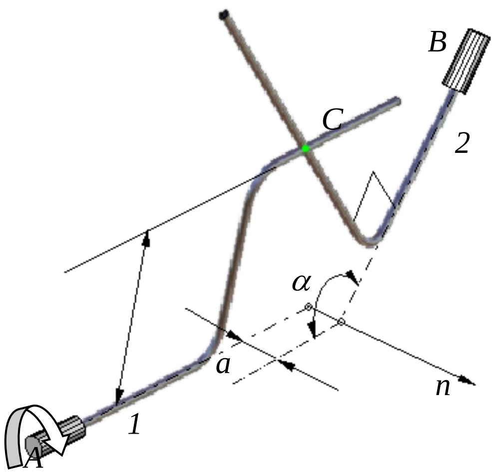

Figure 1 *Two shafts with crossed axes*

It supports the hypothesis that the transversal dimensions of the shafts are much smaller than the length of branches, so the point contact is at the intersection of axes, forming a class 1 pair, as it is shown in the first part of the paper. The relative position of the axes of revolute pairs is characterized by the length of common normal a and the angle between them α. Assuming that the position and motion of the driving element 1 are known, determining the position and displacement of the driven element is required.

2. Proposal of solution

For the kinematic analysis of spatial mechanisms, Hartenberg and Denavit [1] have proposed a method wherein a matrix calculation is used, method called "the method of homogeneous operators". The principle of the method consists in expressing the transformation of the coordinates of a point when passing from a frame to other frame. Basically, the coordinates of a point are passed successively through all possible reference systems placed on the elements of the mechanism until it reaches the original system. By putting the condition of identity between the initial and final coordinates of the point considered, the matrix equation for closing the kinematic chain it is obtained, equation which enables the determination of the positional parameters of the kinematic chain. The method is applicable to all mechanisms that have in their structure cylindrical, revolute or prismatic pairs. In addition, the two authors [1] show that conveniently directing the axes of the reference system, the relative position of two successive frames is not described by six parameters, as in the general case, but only by four parameters. In order to apply the method, the axis Oz must coincide with the axes of the pairs and Ox axes are oriented along to the common normal of two successive axes Oz placed on neighboring elements. The presence of higher pair in the structure of the mechanism to be analyzed makes impossible the direct application of the Hartenberg Denavit method. For the application of the method, it is necessary to replace the higher pair with a kinematical chain, its structure having only the lower pairs. There are several possibilities for replacement. One of these is shown in Fig. 2 and involves the use of a replacement chain consisting of two elements interconnected by a revolute pair. The two elements are related to the two shafts by cylindrical pairs, respectively.

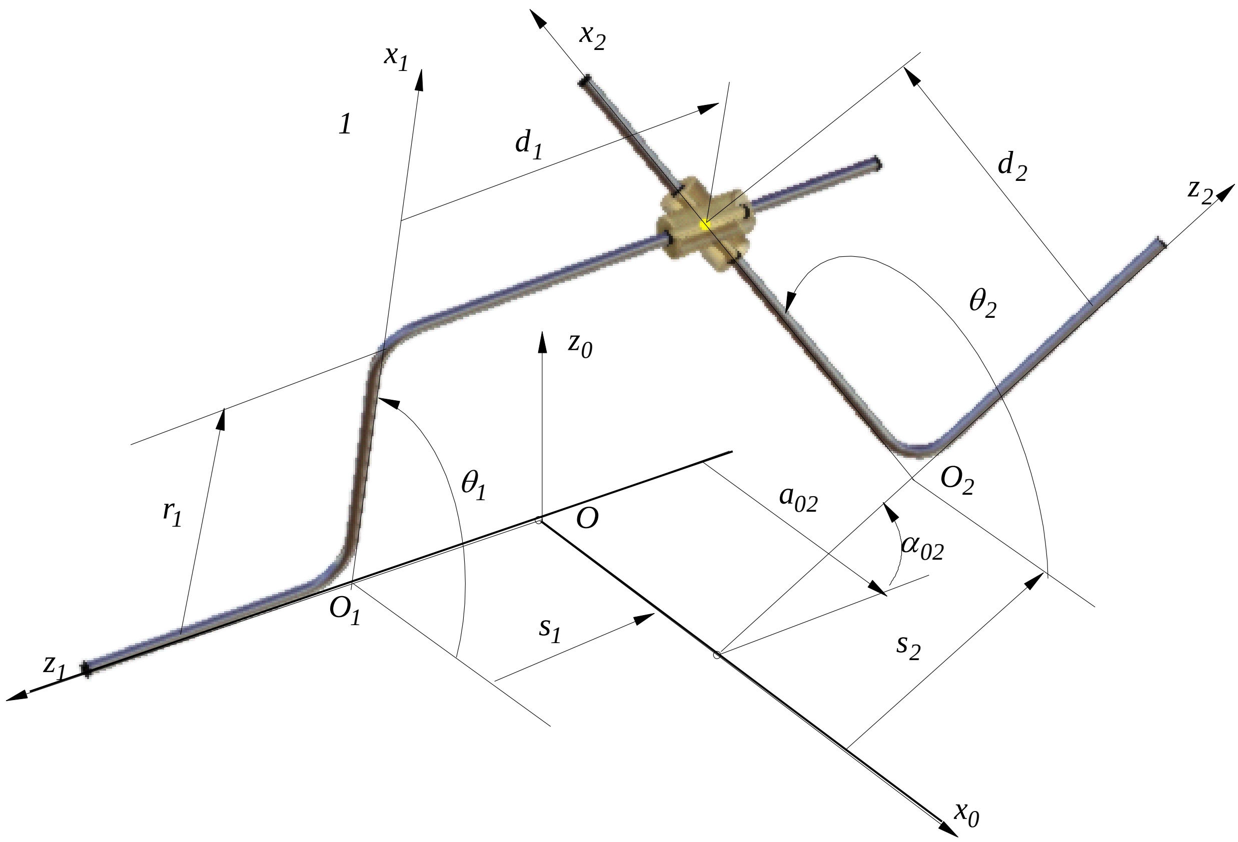

Figure 2 *Geometric and kinematical parameters of the transmission*

In Fig. 2 are represented the two shafts having rotation axes denoted z_1 and z_2. The common normal of these two axes of rotation has been chosen as the axis x_0 of the fixed element (ground). The axis z_0 has been selected the vertical passing through the foot of common normal on the driving axis. The axes x_1 and x_2 have their orientation specified on Fig.2. All geometrical and kinematical parameters must be considered with signum, the signum being established in accordance with the positive direction of the axis about they are determined. It is considered that the driving element has the position determined by the angle θ_1 and element 2 by angle θ_2. The position of the contact point C it is specified using parameters d_1 and d_2. All the other parameters from Fig. 2 are considered constant and known.

3. Conditional equations

The application of Denavit-Hartenberg method for the entire mechanism is laborious and in addition lead to a set of parameters without immediate practical importance, such as the rotations from cylindrical pairs or revolute pairs of the linkage. To avoid this it is used a geometrical condition. This is represented by the contact condition which requires that the points C_1 and C_2 on the two elements to coincide at all times.

$$\boldsymbol{r}_{C_1} = \boldsymbol{r}_{C_2} \tag{1}$$

To be operable, the two vectors in relation 1 must be expressed in the same coordinate system. Below, those two vectors will be expressed in the ground frame For this purpose it is used the observation made by McCarthy [2], which specifies that the transformation of the coordinates of a vector from system q in system s is obtained by the matrix relation:

$$\boldsymbol{x}_s = \boldsymbol{T}_{sq}\boldsymbol{x}_q \tag{2}$$

where $\boldsymbol{x}_r$ and $\boldsymbol{x}_q$ are the point coordinates in those two frames and $\boldsymbol{T}_{sq}$ is the displacement operator which brings the system $"s"$ over system $"q"$. The displacement operator can be expressed as a series of successive displacements in about the known axis. Choosing the coordinate system of axis according to Hartenberg -Denavit convention it allows any displacement $\boldsymbol{T}_{sq}$ to be expressed as a succession of roto-translations about the axis x or axiz z. Aiming that the transformation relation has a homogeneous character, equation 2 is written as:

$$\begin{bmatrix} x_s \\ y_s \\ z_s \\ 1 \end{bmatrix} = \begin{bmatrix} \boldsymbol{R}_{sq} & \boldsymbol{d}_{sq} \\ \boldsymbol{0} & 1 \end{bmatrix} \begin{bmatrix} x_q \\ y_q \\ z_q \\ 1 \end{bmatrix}, \tag{3}$$

where $\boldsymbol{R}_{sq}$ is the rotation matrix of the frame s over frame q, $\boldsymbol{d}_{sq}$ is the position vector of the origin of the frame q with respect to the frame s, and $\boldsymbol{0}$ is the vector:

$$\boldsymbol{0} = [0 0 0] \tag{4}$$

The homogeneous operator corresponding to an angle of rotation α and a translation along the length a about the axis Ox is according to [2]:

$$\boldsymbol{X}(\alpha, a) = \begin{bmatrix} 1 & 0 & 0 & a \\ 0 & \cos\alpha & -\sin\alpha & 0 \\ 0 & \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{5}$$

Similarly, the operator corresponding to a rotation angle θ and to a displacement s about z axis has the general form:

$$\boldsymbol{Z}(\theta, s) = \begin{bmatrix} \cos\theta & -\sin\theta & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & s \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{6}$$

The coordinates of the point C_1 in the frame fixed to the driving shaft 1 are:

$$\boldsymbol{x}_1 = [r_1 \;\; 0 \;\; d_1 \;\; 1]^T \tag{7}$$

and bringing the frame 0 over the frame 1 is achieved by transformation of:

$$\boldsymbol{T}_{01} = \boldsymbol{Z}(\theta_1, s_1) \tag{8}$$

Similarly, the coordinates of C_2 in the coordinate frame attached to the driven element is:

$$x_2 = [d_2 \quad 0 \quad 0 \quad 1]^T \qquad (9)$$

while moving the frame $"0"$ over the frame $"2"$ the following conversion was obtained:

$$T_{02} = X(\alpha_{02}, a_{02})Z(\theta_2, s_2) \qquad (10)$$

Contact condition is written in matrix format, [3-4]:

$$T_{01}x_1 = T_{02}x_2 \qquad (11)$$

Matrix equation 11 is equivalent to the system of scalar equations:

$$\begin{cases} d_2\cos\theta_2 - r_1\cos\theta_1 + a_{02} = 0 \\ d_2\cos\alpha_{02}\sin\theta_2 - s_2\sin\alpha_{02} - r_1\sin\theta_1 = 0 \\ d_2\sin\alpha_{02}\sin\theta_2 + s_2\cos\alpha_{02} - d_1 - s_1 = 0 \end{cases} \qquad (12)$$

The system 12 allows for the determination of kinematical parameters θ_2, d_1 and d_2 as function of the positional angle θ_1 of the driving element:

$$\begin{cases} \theta_1(\theta_1) = -a\tan\dfrac{s_2\sin\alpha_{02} + r_1\sin\theta_1}{(a_{02} - r_1\cos\theta_1)\cos\alpha_{02}} + k\pi \\ d_1(\theta_1) = \dfrac{s_2 + r_1\sin\alpha_{02}\sin\theta_1 - s_1\cos\alpha_{02}}{\cos\alpha_{02}} \\ d_2(\theta_1) = -\dfrac{a_{02} - r_1\cos\theta_1}{\cos\theta_2(\theta_1)} \end{cases} \qquad (13)$$

It can be observed that for θ_2 there are two solutions, offset from each other to 180^o. The physical significance of this result is explained by the two possibilities of forming a symmetrical contact about to the axis of the pair of the driven element. The same can be said about the parameter d_2 which takes values of equal module but of opposite sign, for these two possibilities. As an application there are presented the results for the following data:

$\theta_1 = 20^o$, $r_1 = 70$, $s_1 = 50$, $\alpha_{02} = -150^o$, $a_{02} = 50 \;\; s_2 = 30$,

The solutions of system 12 become:

$d_1 = -70.818$, $d_2 = 18.856$, $\theta_2 = -33.199$

The figures below show the variations of parameters θ_2, d_1 and d_2 for a complete rotation of the driving element.

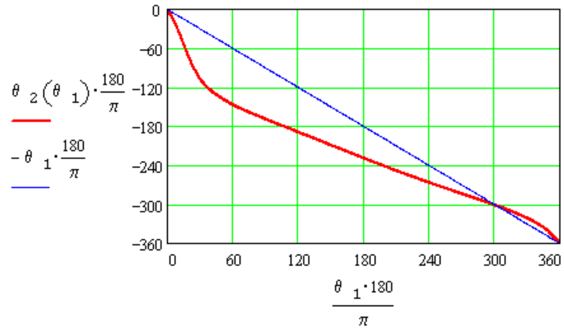

Figure 3 *Driven shaft rotation* θ_2

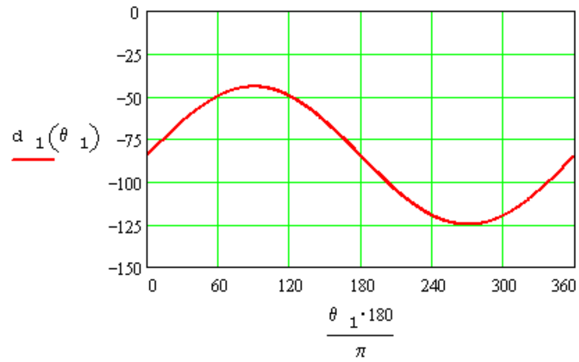

Figure 4 *The displacement of the contact point on the driving shaft* d_1

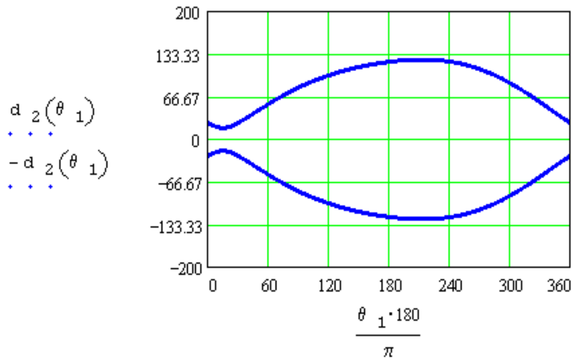

Figure 5 *The displacement of the contact point on the driven shaft* d_2

4. Conclusions

The paper presents a solution for direct coupling of two shafts. The first part shows that structurally the only way for coupling two shafts is the formation of a class 1 pair between the two shafts. By choosing as the concrete way of achieving class 1 pair the contact between two straight lines, the second part of the paper presents the kinematical analysis of the structural solution adopted in the first part. For kinematical analysis, the frames required are adopted according to the methodology established by Hartenberg and Denavit. The direct application of Hartenberg-Denavit method is not possible due to the presence of the higher pair. Even after replacing the higher pair, the method is proving laborious. To overcome this issue in order to solve the kinematics, the condition of contact between the two shafts is employed. Expressing this condition in matrix format leads to a system that allows to obtain directly all variable parameters of the mechanism as functions of the position of the driving element. The application of the method is exemplified for a set of concrete data.

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