THREE DIMENSIONAL VIBRATIONS OF AGGREGATE CONNECTED WITH ELASTIC ELEMENTS

Anastas Ivanov Ivanov

Todor Kableshkov University of Transport, Sofia, BULGARIA, aii2010@abv.bg.

Abstract: *An investigation on the forced small three dimensional vibrations of an asymmetrical body hanging of three mutually perpendicular coil springs is presented in the current paper. The forced vibrations are caused by harmonic changing excitation force. The three dimensional behavior of the coil springs are taken into consideration. Dissipation from internal friction in all elastic elements is reduced by corresponding coefficients of hydraulic friction. All analytical equations in the study are presented in a matrix form. Amplitude frequency and phase frequency characteristics for all generalized coordinates are defined. The numerical solution is done by a special program in the MatLab environment. The study is intended for specialists in the field of vibrations and protection from vibrations.*

Keywords: *3D vibrations, protection from vibrations, amplitude and phase frequency characteristics, matrices, MatLab*

1. Introduction

Three dimensional vibrations of the rigid body, or a system of rigid bodies, connected with elastic elements are studied in a large number of published scientific papers in world literature, for example [Ganiev 1976], [Tcherneva 1987], [Buchvarov 1994], and others. However, the three dimensional behaviour of the elastic element, such as a coil spring, a rubber vibration isolator, metal-rubber elastic element and others, are not accounted accurately in these works. For example, a real elastic element with three coefficients of elasticity is modeled. This is inaccurate and incomplete. It is not allowed us to study the actual free and forced vibrations, particularly from kinematic excitations, which are founded in machines, aggregates and vehicles.

The established theory underlied in the treatise [Ganiev 1976] is correct if it is assumed that all elastic elements are subjected into tension and compression. In many machines, especially in the spring suspension of transport vehicles, such as railways, heavy trucks, planes and other, elastic elements are in a very complicated stress and deformable state. With their considerable sizes such elastic elements can not be replaced by three springs, which are oriented in three mutually perpendicular directions. Their stress and deformable state are three dimensional and not only a tensile (tension) or a compression (pressure). It is not acceptable a real spring element, which has an elastic matrix with dimensionality 12×12, and which connected to the bodies by two large contact areas, with three linear imaginary springs to be replaced. This is obviously inccorect.

Coil spring by Finite Elements Method (FEM) in the work [Ivanov, 1996] was studied. The full stiffness matrix of this spring was obtained and all elastic coefficients, which its stress and deformable state are defined.

An influence of mechanical and geometrical parameters on measuring and control of cylindrical springs in a specialized laboratory of Todor Kableshkov University of Transport is studied. This is published in the paper [Shtarkalev, 2014]. The researchers in this work obtain the stiffness matrix of such springs by an experimentally way.

Currently, many mechanical systems are studied successfully by powerful programs on FEM [Ivanov, 2010]. For example, one such great program package is Ansys [Ansys, 2009]. These programs, however, are very difficult to learn and they give not to the researchers necessary freedom, especially when they describe the excitations and nonlinearity.

The author of this paper offers all elastic and damping elements, which to a rigid body are connected, to be solved by FEM with smaller programs like Solid Edge, Solid Works, Cosmos, Comsol and other [Solid Edge, 2012]. Of course, this approach does not exclude the possibility these elastic elements to be explored experimentally in specialized laboratories. So, accurate information on the elastic and damping characteristics of the spring elements could be obtained and comnfirmed. Moreover, mass and inertia properties of the solid body could be defined numerically by such of these programs too.

Further, the study of small three dimensional vibrations continues with obtaining of the corresponding differential equations. These equations are solved analytically, if they are linear, or numerically by means of specialized mathematical program such as MatLab, MathCAD, Mathematika, MuCAD and others.

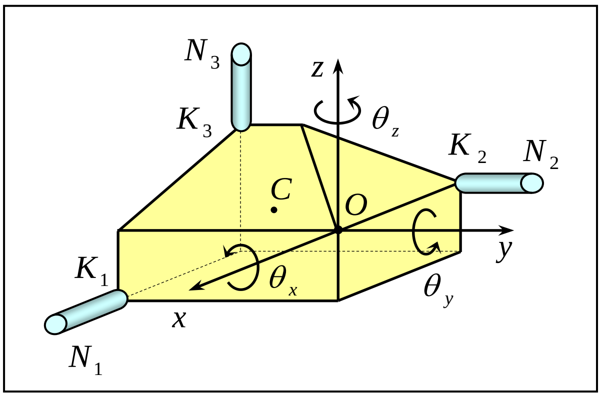

Figure 1: *Dynamical model*

The purpose of this work is the forced small three dimensional vibrations of an aggregate connected to three elastic elements to be studied by the method described above using the previously obtained stiffness matrix and damping matrix.

3. Mathematical model

To illustrate the saying above the following numerical example is solved. A dynamical model of an aggregate is considered. He's not a real machine, but all parameters are selected appropriately and they correspond to the actual event.

The aggregate is treated as an non-homogeneous assymetrical rigid body with a mass $m[kg]$. Its motion is measured to the coordinate system $Oxyz$. For convenience, it is not placed in the center of gravity of the body - Figure 1. The body is connected with three equal rubber-metal elastic and damping elements which along the corresponding axes Ox, Oy and Oz are orientated.

The vector of generalized coordinates that are defined the position of the body during its small three dimensional vibrations, has the form:

$$\mathbf{q} = \left\langle x \quad y \quad z \quad \theta_x \quad \theta_y \quad \theta_z \right\rangle^T, \tag{1}$$

where the functions $x(t)$, $y(t)$ and $z(t)$ define the linear displacements of the pole O, and the functions $\theta_x(t)$, $\theta_y(t)$ and $\theta_z(t)$ define the body rotation around the corresponding axes x, y and z.

The mass and inertia properties of the body with the following symmetrical matrix is determined:

$$\mathbf{A} = \begin{bmatrix} \mathbf{M} & \mathbf{S}_C^T \\ \mathbf{S}_C & \mathbf{J} \end{bmatrix}. \tag{2}$$

It is composed by the following sub-matrices:

I. Diagonal mass matrix,

$$\mathbf{M} = \begin{bmatrix} m & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & m \end{bmatrix}; \tag{3}$$

II. Reverse symmetrical tensor from the first range of the mass static moments,

$$\mathbf{S} = m.\begin{bmatrix} 0 & -z_C & y_C \\ z_C & 0 & -x_C \\ -y_C & x_C & 0 \end{bmatrix}; \qquad (4)$$

III. Symmetrical tensor from the second range of the mass and inertia moments,

$$\mathbf{J} = \begin{bmatrix} J_x & -J_{xy} & -J_{xz} \\ -J_{yx} & J_y & -J_{yz} \\ -J_{zx} & -J_{zy} & J_z \end{bmatrix}. \qquad (4)$$

Rubber-metal elastic and damping elements are connected with rigid body and the immovable ground hardly in the area with centers, respectively K_k and N_k, $(k = 1,2,3)$. The elastic and damping elements as condensation finite elements are modeled. We will consider the two points K_k and N_k joints, which nodal displacements and nodal forces are defined on the Figure 2.

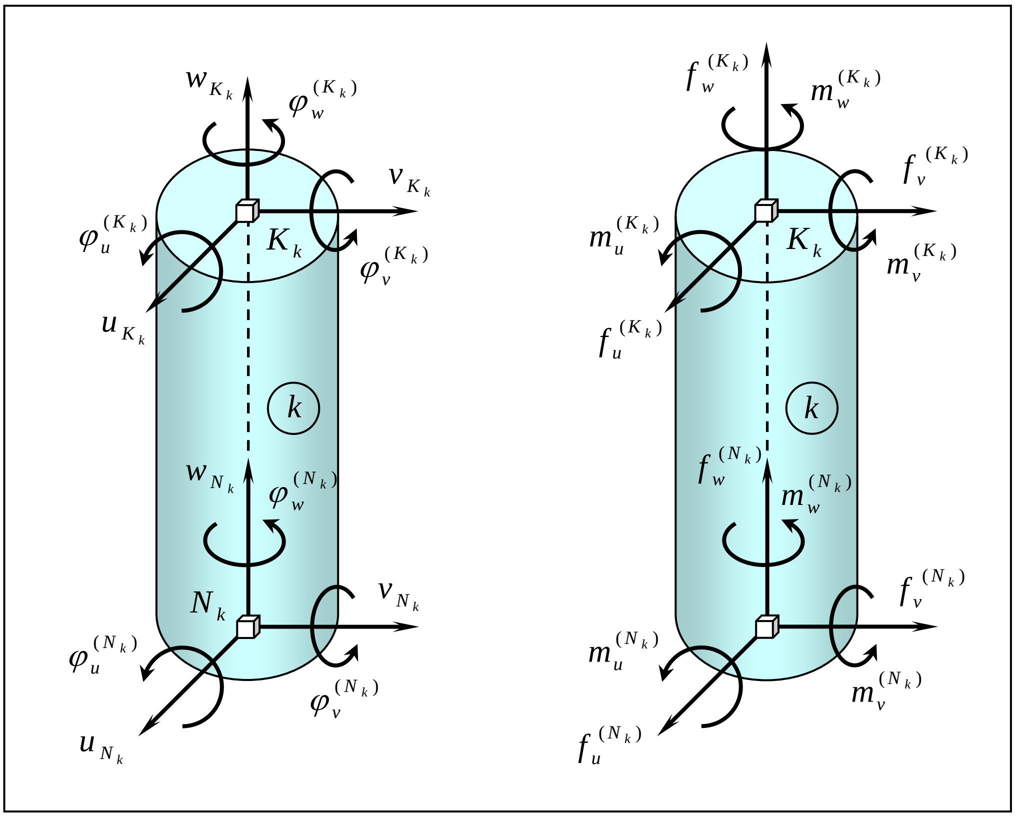

Figure 2: *Joint displacements and joint forces of the elastic elements*

Each of the three elastic elements is defined with a symmetrical matrix of elasticity $\mathbf{C}_k = \left[c_{i,j}^{(k)} \right]_{12\times 12}$, $(k = 1,2,3)$.

The system differential equations describing small forced three dimensional vibrations of the body, as a result of harmonic force effect has the form:

$$\mathbf{A}.\ddot{\mathbf{q}} + \mathbf{B}.\dot{\mathbf{q}} + \mathbf{K}.\mathbf{q} = \mathbf{Q}\ . \qquad (5)$$

In this system of equations the symmetrical stiffness matrix is formed by the formulae:

$$\mathbf{K} = \sum_{k=1}^{3} \mathbf{T}_k.\mathbf{C}_k.\mathbf{T}_k^T = \left[k_{i,j} \right]_{6\times 6}, \qquad (6)$$

where matrices $\mathbf{T}_k$ include dates for applied points K_k and orientations of the elastic elements.

The general force from the right part of the equations (5) is presented by the equal:

$$\mathbf{Q}(t) = \hat{\mathbf{H}}\,.\sin\left(p.t + \mu \right), \qquad (7)$$

where the vector $\hat{\mathbf{H}}$ includes the components of the main force and the main moment of the excitation harmonic force after making a reduction to the coordinate pole O, $p\,[rad/s]$ is the excitation circle frequency of the this force, and $\mu\,[rad]$ is its initial phase.

The matrix $\mathbf{B}$ is also symmetrical and it is defined dissipation properties of the system. With other words, it accounts the inner friction in the rubber-metal elastic elements.

4. Numerically solution

A numerical solution of the system of differential equations (5) with software program MatLab is implemented. The following parameters are used:

Mass of the body: $m = 205.176\ kg$.

Coordinates of the mass center:

$x_C = -0.624$; $y_C = -0.502$;

$z_C = -0.226\ [m]$.

Mass inertia moments: $J_x = 66.289$;

$J_y = 91.643$; $J_z = 32.853$, $J_{xy} = 66.194$,

$J_{xz} = 28.935$, $J_{yz} = 23.973\ [kg.m^2]$

For numerical solution various stiffness matrices and various damping matrices are used.

The base values of non-zero elements of the stiffness matrix are:

$$k_{01,01} = 50250 = k_{02,02} = k_{03,03}\ [N/m];$$
$$k_{02,04} = 19500 = -k_{01,05};$$
$$k_{01,06} = 49700 = -k_{03,04};$$
$$k_{03,05} = 59600 = -k_{02,06}\ [N.m/rad];$$
$$k_{04,02} = 19500 = -k_{05,01};$$
$$k_{06,01} = 49700 = -k_{04,03};$$
$$k_{05,03} = 59600 = -k_{06,02}\ [N/m];$$
$$k_{04,04} = 57500; \quad k_{05,05} = 79400;$$
$$k_{06,06} = 120600; \quad k_{04,05} = -58900 = k_{05,04};$$
$$k_{04,06} = -23600 = k_{06,04};$$
$$k_{05,06} = -19600 = k_{06,05}\ [N.m/rad].$$

The base damping matrix is assumed proportional to the stiffness matrix according to the Voight hypothesis [Seculic, 2011], namely:

$$\mathbf{B} = \delta.\mathbf{K}\ , \tag{8}$$

where the constant δ is appropriate chosen.

The vector of amplitude values of the excitation force has the following values:

$$\hat{H}_1 = 80;\ \hat{H}_2 = 111.8034;\ \hat{H}_3 = 60\ [N];$$
$$\hat{H}_4 = -33.541;\ \hat{H}_5 = 37.818;$$
$$\hat{H}_6 = -66.657\ [N.m].$$

Through the composed program in the area of MatLab the amplitude characteristics for all generalized coordinates of the dynamical model are received:

$$\hat{\mathbf{q}} = \left[\left(\mathbf{K} - p^2.\mathbf{A}\right) + i.p.\mathbf{B}\right]^{-1}.\hat{\mathbf{H}}\ , \tag{8}$$
$$i = \sqrt{-1}\ .$$

Calculations are performed for different values of the stiffness matrix and damping matrix. The most important relations between these matrices are the following:

$\mathbf{K}$ and $\mathbf{B} = 0.05\,\mathbf{K}$, (Figure 3);
$\mathbf{K}$ and $\mathbf{B} = 0.10\,\mathbf{K}$, (Figure 4);
$\mathbf{K}$ and $\mathbf{B} = 0.15\,\mathbf{K}$, (Figure 5);
$1.50\,\mathbf{K}$ and $\mathbf{B} = 0.10\,\mathbf{K}$, (Figure 6);
$1.50\,\mathbf{K}$ and $\mathbf{B} = 0.10\,\mathbf{K}$, (Figure 7);
$3.00\,\mathbf{K}$ and $\mathbf{B} = 0.10\,\mathbf{K}$, (Figure 8).

Graphs of the calculated amplitude characteristics are shown below.

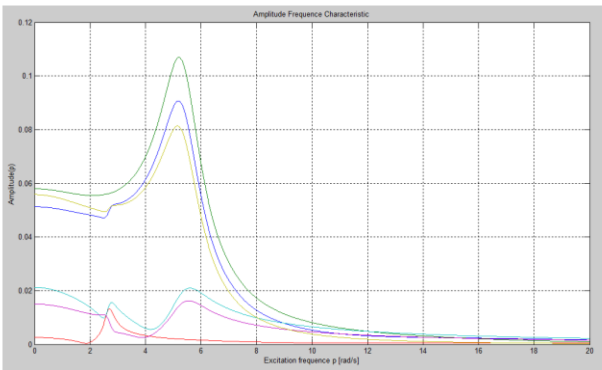

Figure 3: *Ampl.-Ch. for* $\mathbf{K}$ *and* $\mathbf{B} = 0.05\mathbf{K}$

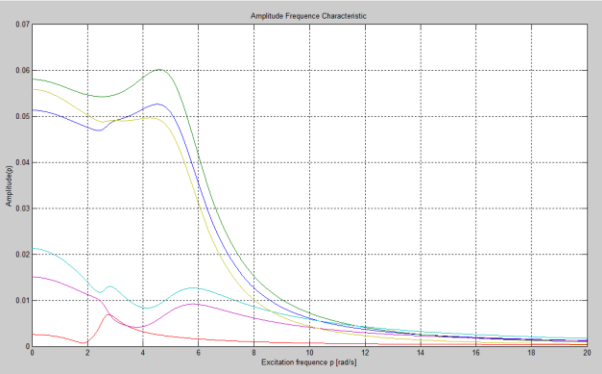

Figure 4: *Ampl.-Ch. for* $\mathbf{K}$ *and* $\mathbf{B} = 0.1\mathbf{K}$

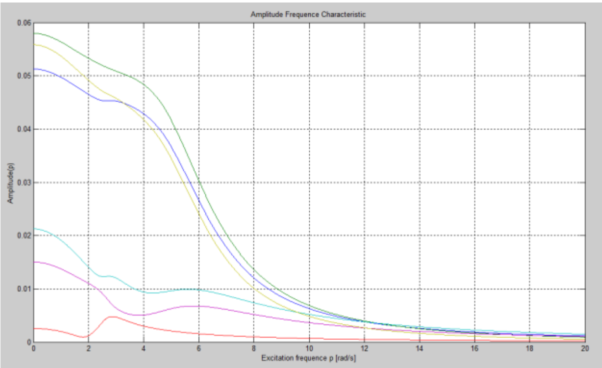

Figure 5: *Ampl.-Ch. for* $\mathbf{K}$ *and* $\mathbf{B} = 0.15\mathbf{K}$

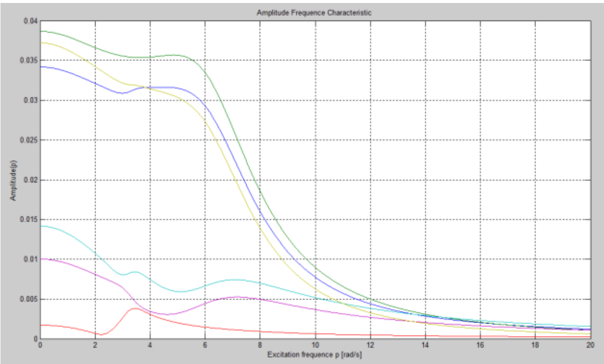

Figure 6: *Ampl.-Ch. for* $1.5\,\mathbf{K}$ *and* $\mathbf{B} = 0.1\mathbf{K}$

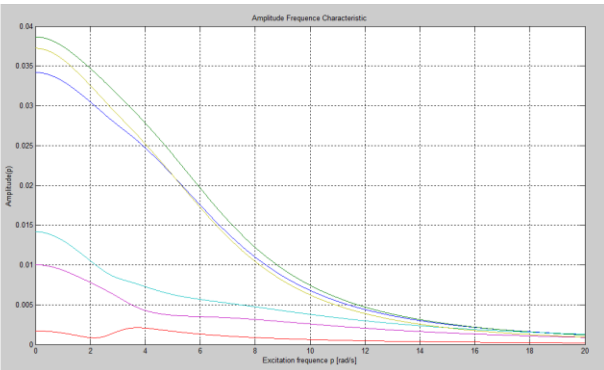

Figure 7: *Ampl.-Ch. for* $1.5\,\mathbf{K}$ *and* $\mathbf{B} = 0.2\mathbf{K}$

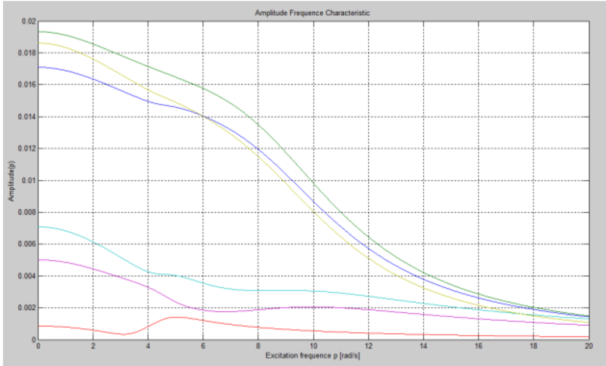

Figure 8: *Ampl.-Ch. for* $3\,\mathbf{K}$ *and* $\mathbf{B} = 0.1\mathbf{K}$

5. Results and discussions

Eigen circular frequencies, determined for the stiffness matrix $\mathbf{K}$, (Figures 3, 4 and 5) are: 2.7; 5.4; 5.4; 15.6; 45.4; 161.9 $[rad/s]$.

Resonance increases in the amplitudes of all coordinates in the zones around eigen circular frequencies are noticed. This can be seen particularly in Figure 3, a little less in Figure 4, and at least in Figure 5. All amplitudes shown in Figures 3, 4 and 5 start from the same initial position. The above three graphics refer to the amplitudes on directions x, y and z. The bottom three graphics refer to the amplitudes of rotations θ_x, θ_y and θ_z.

By consequently increasing the damping, namely $\mathbf{B} = 0.05\,\mathbf{K}$, $\mathbf{B} = 0.10\,\mathbf{K}$ and $\mathbf{B} = 0.15\,\mathbf{K}$, there is a blunting of the resonance peaks around eigen frequence zones. Using the stiffness matrix $\mathbf{K}$ and damping matrix $\mathbf{B} = 0.15\,\mathbf{K}$ the most smooth amplitude frequency characteristics are obtained.

Eigen circular frequencies, determined for the stiffness matrix $1.50\,\mathbf{K}$, (Figures 6 and 7) are: 3.3; 6.6; 6.6; 19.2; 55.6; 198.3 $[rad/s]$.

Here some resonance amplitude increases for all coordinates by using the damping matrix $\mathbf{B} = 0.10\,\mathbf{K}$ are noticed. With the stronger damping, namely matrix $\mathbf{B} = 0.20\,\mathbf{K}$, there is a slight amplitude increase only at amplitude of rotation θ_z.

This is the bottom curve in Figure 7. For all other coordinates a smoothly decrease the amplitudes of the whole frequency area is noticed.

Eigen circular frequencies, determined for the stiffness matrix $3.0\,\mathbf{K}$, (Figure 8) are: 4.6; 9.3; 9.3; 27.1; 78.6; 280.5 $[rad/s]$.

Here nature of all curves remains as in Figure 7 with one difference that the initial amplitudes start twice as low.

For all the charts is established that increasing the stiffness the values of initial amplitudes decrease. And increasing the damping the resonance peaks are reduced and even disappear.

In depending of the project specification the engineer has to select such a ratio between stiffness and damping that the amplitudes in all generalized coordinates to be less than the limit. But at the same time is necessary the resonance peaks around all eigen circular frequencies to be missing.

6. Conclusion

A modern methodology for modeling and analysis of the three dimensional vibrations of the rigid body, connected with elastic and damping elements is presented.

Geometrical mass and inertia characteristics are determined by programs for 3D modeling and design.

The characteristics of the elastic elements can be defined by two ways. The first of them represents a modeling with programs working on the Finite Element Method. A full stiffness matrix of the elastic elements is obtained. It depends on the type of elements and their connections with other bodies.

The second approach is by experiments. The elastic elements, or a small-scaled and simplified model of them, are tested in a specialized laboratory where all necessary parameters, as well as the stiffness matrices, are determined and retrieved.

The characteristics of the damping elements are determined in a similar way as the elastic ones. If they are separated from the elastic elements as independent units, their characteristics are given by the manufacturers with the whole product. If there are not such damping characteristics, from one hand, or if they are caused by the inner friction, to the other hand, it is better these characteristics in the specialized laboratory to be received. Based on records of the closed hysteresis curves the equivalent viscous resistance can be obtained.

Further the type of excitation is determined and the differential equations, describing the body motion, are carried out. Finally, a solution with specialized mathematical program as MatLab, MathCAD, MuCAD and so on is done.

References

[1] Ganiev R.F., Komonenko V. O., *Vibrations of rigid bodies*, Nauka, Moscow, 1976 (in Russian).

[2] Cherneva Z., *Applied theory of vibrating protection*, Technical University, Sofia, 1987 (in Bulgarian).

[3] Bachvarov S., Cherneva Z., Banov S., *Vibrations, protection from vibrations and noises in machiones*, Technical University, Sofia, 1987 (in Bulgarian).

[4] Ivanov A.I., Chobanov V. N., *Investigation of elastic properties of cylindrical springs by FEM*, Intern. Scientific Conference VSU'1996, 20-21.11.1996, Proc., vol. 2.

[5] Shtarkalev I. B., Paskalev L. K., *Rating the influence of mechanical and geometrical parameters on measuring and control of cylindrical springs*, Machanics, Transport, Communication, vol. 12, issue 3/3, pp. VI 11-18, Sofia, 2014, (in Bulgarian).

[6] Ivanov A.I., *Structural Dynamics*, Avangard Prima, Sofia, 2010, (in Bulgarian).

[7] Ansys, *Multibody Analysis Guide*, Ansys, Inc.. Southpointe 275 Technology Drive, Canonsburg, PA 15317, Release 12.1, 2009

[8] Solid Edge, *Solid Edge assembly*. Mt 01416-s-1050, Siemens Product Lifecycle Management Software Inc., 2012.

[9] Seculic D., Dedovic V., *The effect of stiffness and damping of the suspension system elements on the optimization of the vibrational behaviour of a bus*, IJTT, 2011, 1(4): 231-244.