UPON EXPERIMENTAL DATA INTERPOLATION USING POWER FUNCTIONS. PART I: THEORETICAL CONSIDERATIONS

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Abstract: *The first part of the paper presents e few considerations concerning the interpolation of experimental results with the aid of power functions. For a data raw, generated using a power law function, the effect of origin displacement upon the accuracy of finding the parameters of power law interpolation function is analyzed.*

Keywords: *interpolation, power law function, origin displacement*

1. Introduction

In experimental research, often occurs the problem of estimating the influence that some parameters produce upon other characteristic. Generally, it is difficult to establish the simultaneous effect of all parameters upon the parameter studied and it is preferably to study separately the influence of each parameter upon the dependant variable.

At the closing stages of the experimental tests, a dependency between the dependant characteristic and the independent parameter is obtained in a table form. To draw some general conclusions upon the studied relationship, it is required that the experimental data should be interpolated. The interpolation procedure has two main goals: 1) the possibility of estimating the behaviour of the studied system for the values of independent parameters from the range used in experimental tests (the interpolation in fact); 2) the option of anticipating the behaviour of the studied system for values situated outside the range of known data points (extrapolation).

The interpolation operation presents two alternatives. The first variant assumes finding a curve that passes through all experimental points (interpolation through points). As interpolation functions, there are usually applied polynomial functions, Newton’s interpolation polynomials, Lagrange interpolation polynomials, [1], spline functions [2], and trigonometric polynomials [3].

There are some drawbacks of the method and there can be mentioned the large number of constants occurring in the interpolation functions, a number equal to the number of pairs of interpolation points. Another disadvantage of this interpolation variant consists in the fact that all the errors from experimental results will be transferred to the interpolation function. The disadvantages of interpolation through points are eliminated by the interpolation between points.

In this case it is aimed a curve that best fits the experimental points. The term “best” assumes finding the expression for interpolation function and the optimum criterion. Choosing the interpolation function supposes that the researcher has both intuition and experience. An optimum criterion may be expressed by the condition that the sum of absolute values of the distances from the experimental points to the interpolation curve should be minimum.

Another criterion used more, requires that the sum of the squared distances from experimental points to interpolation curve ought to be minimum, method known as the least squares method. Regarding the interpolation function from, it is desired that the number of constants involved in the expression to be minimum. Using a large number of constants in the structure of interpolation function has as effect the embedding of experimental errors.

To support this affirmation, it can be mentioned the fact that one of the most utilized software for data processing MICROSOFT EXCEL [4], uses as interpolation functions polynomials of maximum degree four. In technical applications, one of the most used interpolation function is the power function. The advantage resulting from employment of this type of function is the small number of constants from its structure (the coefficient and the power exponent) and the possibility of finding it by analytical method.

Some theoretical aspects are presented in the first part of the present paper.

2. Employing power law as interpolation function

The general form of the power function is:

$$y = C x^{\alpha} \tag{1}$$

where C and α are real constants. With given (x_k, y_k) experimental points, applying the least square method assumes minimizing the function:

$$F(C,\alpha) = \sum_{k=1}^{n} [C x_k^{\alpha} - y_k]^2 \tag{2}$$

The minimum condition for function (2) leads to the system:

$$\begin{cases} \dfrac{\partial F}{\partial C} = 0 \\ \dfrac{\partial F}{\partial \alpha} = 0 \end{cases} \tag{3}$$

To find the parameters C and α, the system of equations (3), which is a transcendent system must be solved and to encompass this aspect, the logarithmic function is applied to relation (1):

$$\ln y = \ln C + \alpha \ln x \tag{4}$$

By denoting $X = \ln x$ $Y = \ln y$ and $\beta = \ln C$ establishing the interpolation function assumes finding the parameters of the straight line:

$$Y = \alpha X + \beta \tag{5}$$

Most of the software programs, such as MATHCAD, Matlab, [5], have dedicated subroutines and it is sufficient to precise the vectors ***x***, ***y*** that contain the coordinates of experimental points.

Nevertheless, a series of particularities concerning the effect of experimental points choice upon the values of C and α constants. To exemplify this affirmation, it is considered a row of points, generated with the function:

$$y = A x^{\beta} \tag{6}$$

For the parameters' values $A = 20$ and $\beta = 2$, and x taking values in the range $[0, 10]$. The domain was divided into $n = 100$ equal intervals. The points (x_k, y_k) are found at the borders of sub-intervals using the relation (1). A first question refers to the manner the number of selected points influences the precision of finding the parameters of interpolation function.

To this purpose, there were chosen the points starting with $x_{k_{in}}$ to x_{k_f}, Fig. 1. Considering that $k_{in} = 1$ for different values of k_f there were found the parameters C and α of interpolation function, using the logarithmic representation (4) of interpolation function. The results are presented in Fig. 2 and Fig. 3.

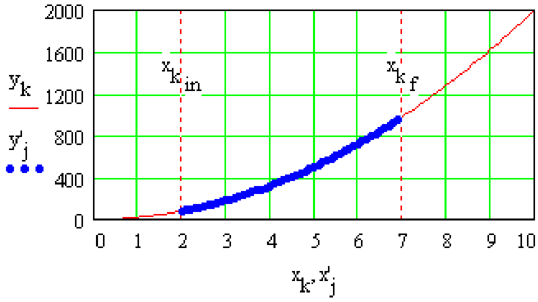

Figure 1 *Graphical representation of interpolation function and the experimental points*

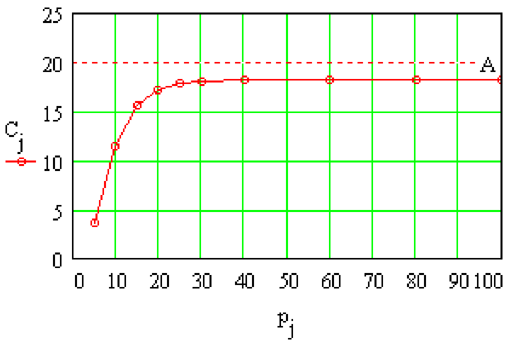

Figure 2 *The dependency of interpolation coefficient on the number of experimental points*

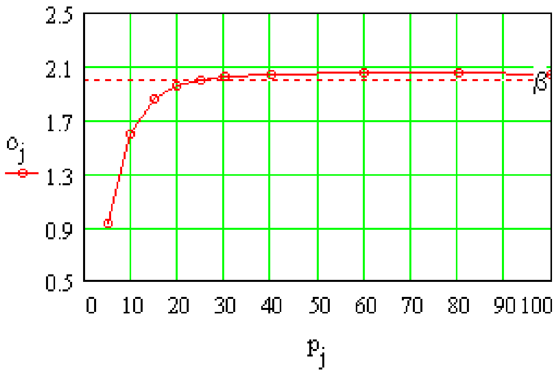

Figure 3 *The dependency of exponent value from interpolation function versus the number of experimental points*

Fig. 2 and Fig. 3 show that, when the function is expressed in double logarithmic coordinates, reducing the number of experimental points influences strongly the parameters of interpolation function.

In the case of using Cartesian coordinates for the interpolation function, the properties of interpolation function are much better "preserved". It was noticed that using only the first six experimental points allows finding accurately the parameters of interpolation function.

The problem arising yet is solving the system of transcendent equations (3), and particularly the difficulty of setting the "guess value".

To surmount this aspect, the concrete form (7) of system (3) it is analyzed and it can be noticed that both relations can be solved with respect to C:

$$\begin{cases} \sum_{k=k_{in}}^{k_f} [C x_k^{\alpha} - y_k] x_k^{\alpha} = 0 \\ \sum_{k=k_{in}}^{k_f} [C x_k^{\alpha} - y_k] x_k^{\alpha} \ln x_k = 0 \end{cases} \tag{7}$$

Thus, the first equation from system (7) is solved for C and the expression obtained is introduced into the second equation (7). By denoting:

$$h(\alpha) = \sum_{k=k_{in}}^{k_f} \left[\frac{\sum_{k=k_{in}}^{k_f} x_k^{\alpha} y_k}{\sum_{k=k_{in}}^{k_f} x_k^{2\alpha}} x_k^{\alpha} - y_k \right] x_k^{\alpha} \ln x_k \tag{8}$$

solving the system is reduced to working out the transcendent equation:

$$h(\alpha) = 0 \tag{9}$$

A graphical representation of function variation within a domain where the solution of equation presumably exists, allows finding a value close to the solution of equation. In Fig. 4 it is presented the plot obtained with considered data.

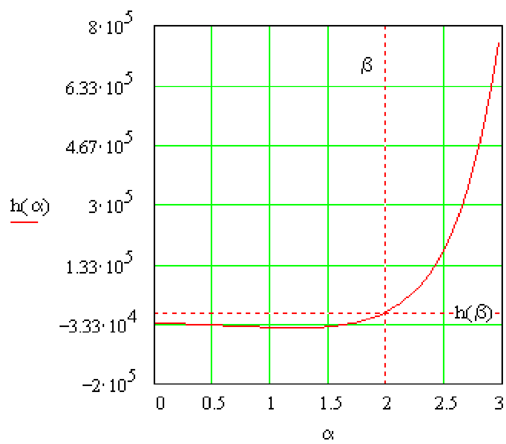

Figure 4 *Graphical estimation for the position of the root of equation (9)*

To be remarked that, for equation (9), the exact value of the solution is found using only five consecutive points regardless of where these points are placed.

3. The consequence of origin displacement upon the parameters of interpolation function

Next, the influence of origin displacement upon the parameters of interpolation function is presented, first for horizontal direction: in Fig. 5, the effect upon exponent and in Fig. 6, the effect upon the coefficient.

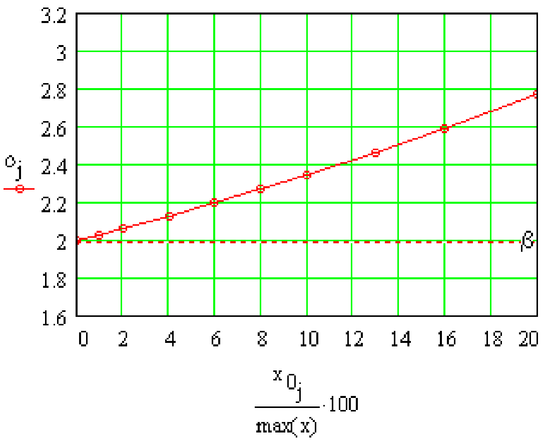

Figure 5 *Dependence between exponent and horizontal displacement of origin*

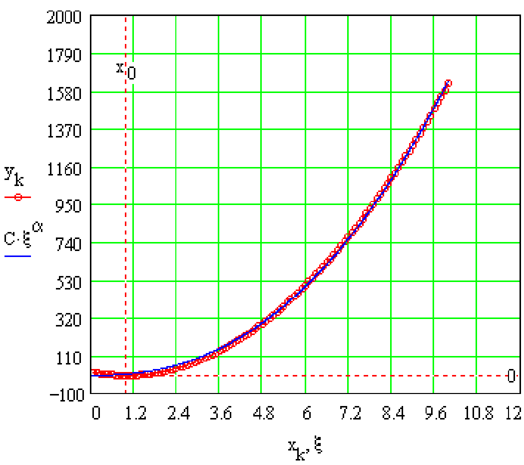

Figure 6 *Experimental data and the interpolation curve for* $x_0 / x_{max} = 0.1$

In Fig. 7 and Fig. 8, there are presented the effects of vertical displacement of origin upon the exponent and the coefficient of interpolation function, respectively.

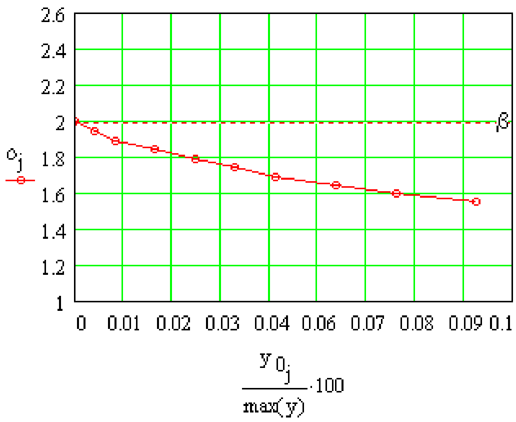

Figure 7 . *Dependency of exponent on the vertical displacement of origin*

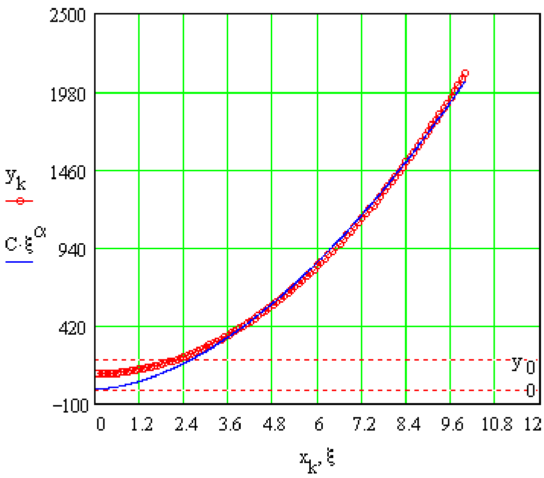

Figure 8 *Experimental data and interpolation curve for* $y_0 / y_{max} = 0.05$

4. Conclusions

In this first part of the paper there is considered a raw of data generated using a power law function. Finding the parameters characteristic to another power function which better describes the data was performed by two methods: the first one, using logarithmic coordinates for the data and the second, keeping the Cartesian form. It was noticed that using the Cartesian coordinate data preserves better the properties of initial function and thus, based on this methodology, it was studied the effect of origin displacement upon the parameters of interpolation function.

The consequences of origin displacement are stronger upon the exponent than upon the coefficient, regardless the direction of displacement.

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