SOME ASPECTS UPON MODELLING THE BEHAVIOUR OF A CORRUGATED MEMBRANE USING FINITE ELEMENT ANALYSIS

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Abstract*: The advantages offered by elastic membranes recommend their use in modern applications. Employment of new materials and constructive restraints require the analysis of membrane behaviours under loading. As the analytical study is often complex, numerical procedures are an alternative. The paper presents two main aspects related to the behaviour of a circular corrugated membrane. Using finite element software the influence of membrane’s thickness upon maximum deflection is analyzed. Natural frequencies were found for several thicknesses of the membrane.*

Keywords: corrugated membrane, finite element analysis, deflection, natural frequency

1. Introduction

Elastic membranes are widely used in technical applications due to the broad range of mechanical characteristics’ variation. The employments of elastic membranes are numerous, from MEMS devices, actuators and pneumatic instruments to biomechanical ventilation systems, as replacement for biological membranes, [1-4].

The main advantages are important deflections without remnant deformations, characteristics very close to linear ones, the possibility of modifying the characteristic by varying the depth of corrugations, great stability. They are used in measuring elements, actuators and controlling devices.

The main disadvantage is related to the fabrication technology, compared to plane membranes the corrugated membranes assuming manufacturing complexity.

To characterize an elastic corrugated membrane, there have to be established the axial profile, the thickness of membrane, the elastic characteristics of the material from which the membrane is made and the fixing system. An important characteristic of a membrane is the ability of supporting reversible deflections under loading. The study of membrane behaviour assumes using an extremely intricate mathematical apparatus, based on the equations of theory of elasticity, [5-10], that guide, for most of the situation, to complicate equations with partial derivates that hardly ever present analytical solutions.

Seldom, in applied engineering, for a rapid design, numerical methods are recalled since for a certain problem they give tangible values of needed parameters. One of the most widespread methods in analysis of deformable bodies is the finite element method, [11-12].

2. Proposed analysis method

To improve the performances but specifically to increase the stiffness of a circular membrane there are made concentric corrugations and for certain depth of corrugation the pressure-deflection characteristic is linear, [13-19]. A circular steel corrugated membrane is considered for the present study, having the profile made of connected circular arcs, Fig. 1, and two aspects are envisaged: the effect of material thickness upon the maximum deflection when a pressure is applied, and finding the natural frequencies. To answer to the first matter, the corrugated diaphragm was modelled using

CATPart module from CATIA software, [20]. Subsequently, using the finite element module of the software, the aspects mentioned above were intended. The membrane is recessed on the boundary using a circular ring. Fig. 2 presents the mesh of discretization used in applying the finite element method. To describe the performance of the membrane as accurate as possible, a mesh with variable size, automatically optimized by the software was implemented.

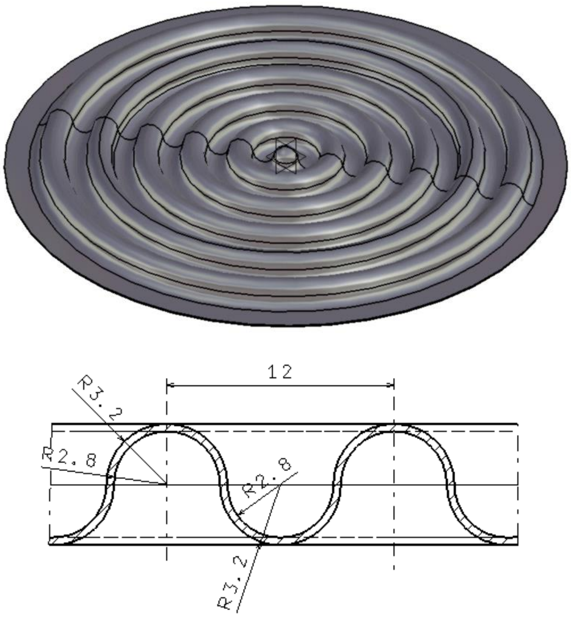


Figure 1 *Circular membrane - axial section*

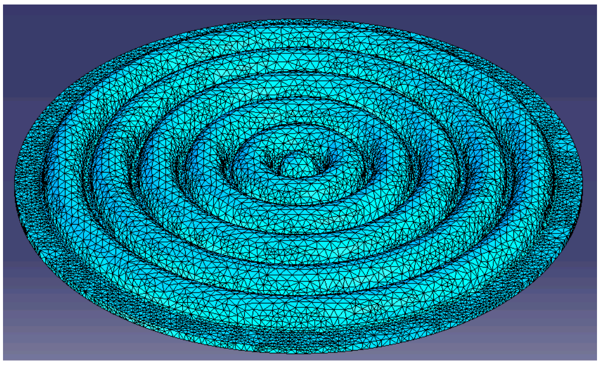


Figure 2 *Membrane discretization with variable mesh size*

In Fig. 3 and Fig. 4 there are presented the deflection and the equivalent von Mises stress of the membrane, respectively. As expected, the maximum deflection occurs in the centre of the membrane but concerning the equivalent stress, the maximum values are obtained about the middle of radial distance. An important problem for a given geometry, known material and précised load, is estimation of maximum deformation. An exact solution of the problem assumes applying the principles from theory of elasticity, [21], but only for extremely simple geometries would be possible to attain an analytical expression of deformation.

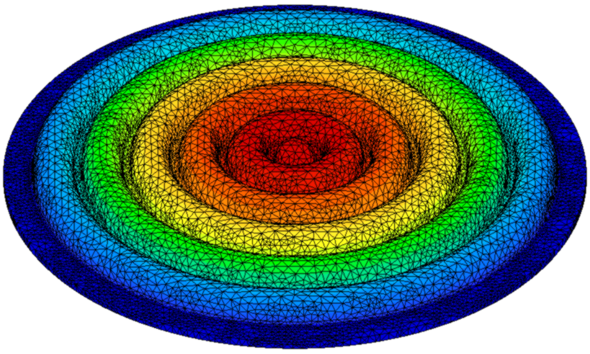


Figure 3 *Deflection of a membrane with constant thickness*

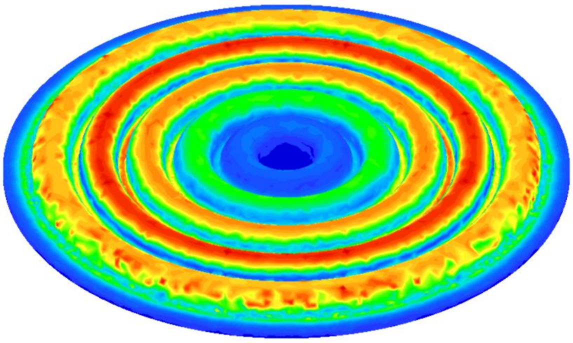


Figure 4 *. Equivalent von Mises stress from a membrane*

To answer to this question, a membrane having the geometry presented in Fig. 1 was considered, with the maximum active diameter $d_{max} = 132mm$, fastened on the boundary, loaded by a constant pressure $p = 10^5 Pa$. Adopting thicknesses of the material in the range $0.1 \div 2mm$, applying the finite element software, the maximum deflection was found, as mentioned, in the centre of the part.

Fig. 5 presents the dependency of maximum deflection on the membrane's thickness. The data obtained from modelling, presented in Table 1, were interpolated with a power function. The interpolation function of data was obtained using the least square method and has the form:

$$y_{max}(\delta)=0.1248^{-0.806\delta} \qquad (1)$$

From Fig. 5 one can observe a strong decrease of maximum deflection with increasing membrane thickness.

Table 1.

δ	0.1	0.2	0.3	0.4	0.5	0.7	1	1.5	2
y_{max} $*10^3$	778	476	346	273	220	152	96	57	38

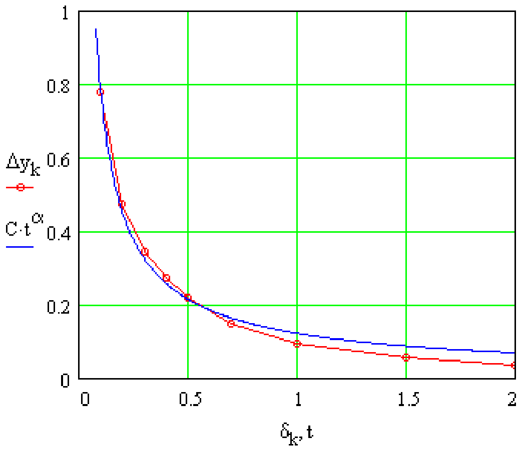


Figure 5 *Maximum deflection versus membrane thickness*

3. Finding the natural frequencies of the membrane

The elastic membranes are often met in complex mechanical systems whose operating conditions involve stationary phases and transition phases, [22]. During transition periods, rapid changes of characteristic parameters of the system take place. A major risk that may occur during running a membrane is the resonance phenomenon happening. The resonance takes place when the frequency of external excitation equals one of the natural frequencies of the membrane, [21]. It is necessary to avoid running in resonance regime because at resonance, the amplitudes of oscillations increase theoretically unlimited and finally the elastic element is broken down. The membrane can be modelled as a deformable solid it will present infinity of natural frequencies. To each of these frequencies a vibration mode corresponds. To pass up the resonance phenomenon, it is required that the smallest natural frequency, the fundamental frequency, should be greater than the excitation frequency. Using the finite element module from CATIA software for three membrane thicknesses, ($\delta=.1;1;2mm$), the first ten frequencies were found after applying the analysis; the results are given in Table 2.

Table 2.

mode number	thickness (mm)		
	$\delta=0.1$	$\delta=1$	$\delta=2$
	frequency (Hz)		
1.	3274	2964	2304
2.	6177	4280	4083
3.	6576	4909	4106
4.	9550	8828	7654
5.	9767	9504	7788
6.	10111	9646	7812
7.	12765	10801	10125
8.	13182	10847	10154
9.	14686	12433	10784
10.	15447	12626	10847

Plotting the three rows of values of natural frequencies corresponding to the three membrane thicknesses prove that for all thicknesses the values of natural frequencies of modes 4, 5 and 6 are in the same vicinity. In Fig. 7 there are presented the deformation modes corresponding to these modes for the $\delta=1mm$ thick membrane.

It can be noticed that the vibration modes 4 and 5 represent, actually, the same vibration but axially de-phased with 90^o, these vibration modes present symmetry with respect to the centre of the membrane.

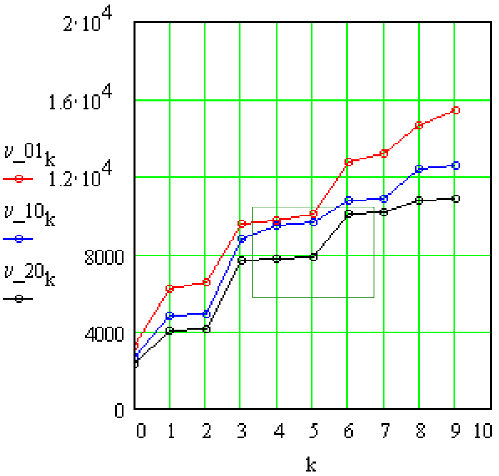
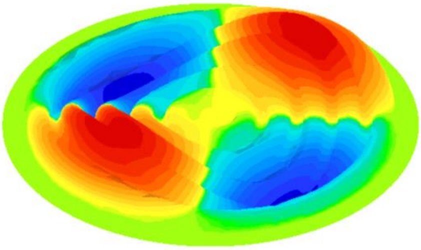
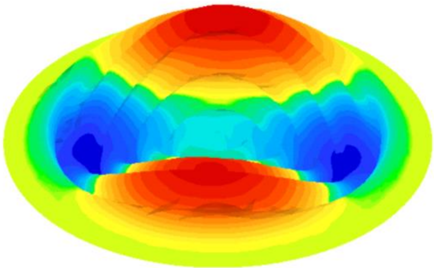


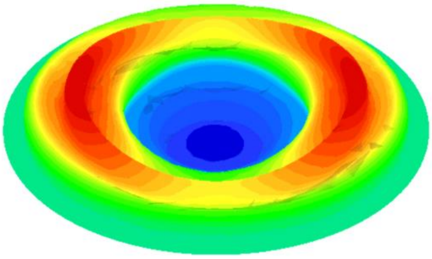
Figure 6 *Frequencies of first ten natural modes*



Mode 4



Mode 5



Mode 6

Figure 7 *Deflections of the membrane for the natural frequencies corresponding to modes 4, 5 and 6, for* $\delta = 1mm$

In a different way compared to these two modes, the six-th vibration mode is symmetric with respect to the axis of the membrane.

4. Conclusions

The paper presents the results obtained by modelling an elastic steel corrugated membrane using CATIA software. The axial profile of the membrane is made of connected circular arcs. The membrane is fixed on the boundary and loaded by a constant pressure.

The finite element analysis is performed used the specialised module of the software and two aspects are aimed: the effect of membrane thickness upon deflection and finding the natural vibration frequencies. It was remarked a strong dependency of maximum deflection on membrane thickness, and the power function approximation of the results gives an exponent of approximate -1. Regarding the values of natural frequencies of vibration modes, the effect of thickness is less important.

Classical design of corrugated membranes is cumbersome and available for a limited number of corrugation types. Modern technical applications suppose a broad constructive variety of elastic corrugated devices. By validating a model analyzed with finite element analysis, the results for other designed models, more complicated, are plausible.

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