THE INFLUENCE OF LOCATION AND STIFFNESS OF SUPPORTS ON THE CHARACTERISTICS OF SPATIAL OSCILLATIONS OF THE BUS POWERTRAIN

Yevhen Kharchenko, Volodymyr Palyukh

Lviv Polytechnic National University, e-mail: kharchen@polynet.lviv.ua, palyukh_v@ukr.net

Abstract: *A mathematical model of the spatial vibration powertrain bus considering geometric nonlinearity of system and gyroscopic system effects has been elaborated. The comparative analysis of the vibration characteristics of the power unit in case of installing it on three, four or five bearings in the working frequency range of loads due to dynamic unbalance elements of the engine has been elaborated*.

Keywords: *mathematic model, power aggregate, oscillation, nonlinear*

1. Introduction

With the development of a fleet of vehicles, as well as wide range of power units that can be mounted on cars and buses, much attention is paid to the vibration and acoustic comfort of drivers and passengers as due to international and national standards, and to maintain health, safe transportation of goods, reliable operation of all machinery and systems of the vehicle. [1–6].

Vibration Active power plant, which significantly affects the mechanical vibrations of the whole system of the vehicle, essentially depends on the mode of the engine. Engine, as a source of vibration, creates the most adverse impact on the city bus, the movement is characterized by the fact that the engine is running at idle more than 1/3 of the total operating time and the fact that the duration of the phases of rapid and slow motion is more than half the total time operation [3, 5, 6, 7].

Vibration power plant due to dynamic unbalance motor elements, mainly researched on the basis of linear, mostly flat computational models [1, 2, 7]. The more accurate - spatial models of vibrating processes powertrains have been proposed in scientific works [5]. Interference of the aggregate oscillations and load carrying structure, which is caused by the movement of a car or bus on the road with irregularities, is analyzed on the basis of simplified chain or flat models [1, 2, 7], because the amplitudes of vertical oscillation of the body and aggregate are the largest in these regimes. The oscillations of the elements of the vehicle in the vertical plane and dynamic phenomena in transmission are considered together in scientific works [1, 2].

Despite the fact that the theory of spatial fluctuations in solids and their systems sufficiently processed [8], the question of selecting the number of poles of the power unit, their characteristics and rational allocation is not fully covered in the literature. At the same time, their solution is a prerequisite for effective design of vehicles.

2. Setting of the Problem

This work concerns the development goal of the mathematical model of spatial fluctuations powertrain bus considering geometric nonlinearity and gyroscopic effects and comparative analysis of vibration characteristics of the power unit in case of installing it on three, four or five poles in the working frequency range of loads caused by dynamic Imbalance elements engine (from 12,732 to 35,014 Hz, corresponding cyclic frequency range from 80 to 220 rad/s).

3. The Mathematical Model of Spatial Oscillations of the Power Aggregate

The power aggregate of the vehicle is considered as a rigid body installed on a stationary basis at *n* elastic supports (Fig. 1). To determine the position of the power aggregate at an arbitrary point in time we use three coordinate systems: a fixed system $Ox_0y_0z_0$, the system always linked to power aggregate $C\xi\eta\zeta$, the beginning of which is located in the center of gravity of the aggregate, and the axis is the principal axis of inertia and the system *Cxyz*, which beginning coincides with the center of gravity of the aggregate, and the axis are moving parallel with respect to the fixed coordinate system.

We assume that the beginnings of the coordinates of all three systems are the same at the initial time.

Spatial oscillations of the power aggregate will be regarded as the result of the imposition of translational motion of its center of gravity *C* and spherical motion around its center. The position of the center of gravity *C* in a fixed coordinate system will be determined by the center coordinates x_c, y_c, z_c, and the position of the aggregate in its spherical movement - using Euler angles ψ, θ, φ (fig.1).

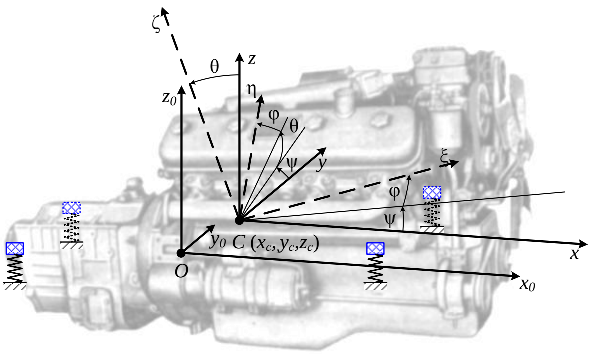

Figure 1: *The coordinate systems for the study of spatial oscillations of the power aggregate*

The equations of translational motion of the center of mass of the power aggregate in a fixed coordinate system can be written as:

$$m\frac{dv_{cx}}{dt}=F_x-\sum_{i=1}^{n}R_{xi};$$
$$m\frac{dv_{cy}}{dt}=F_y-\sum_{i=1}^{n}R_{yi};\quad m\frac{dv_{cz}}{dt}=F_z-\sum_{i=1}^{n}R_{zi},\tag{1}$$

where *m* – the mass of the power aggregate, v_{cx}, v_{cy}, v_{cz} – projection of velocity of the center of mass on a fixed axis Ox_0, Oy_0 i Oz_0; F_x, F_y, F_z - the projection of the main vector of loads caused by dynamic unbalance mechanisms, reduced to the center of gravity of the aggregate on the fixed axes; R_{xi}, R_{yi}, R_{zi} (i = 1, 2, …, 4) – the projection of reactions of elastic supports on a fixed axis; t – time.

For the coordinates of the center of mass of aggregate the fair interrelations are

$$\frac{dx_c}{dt}=v_{cx};\quad \frac{dy_c}{dt}=v_{cy};\quad \frac{dz_c}{dt}=v_{cz}.\tag{2}$$

The equation of spherical motion of the power aggregate around the point *C* a Рівняння сферичного руху силового агрегату навколо точки *C* record in the moving coordinate system *C*ξηζ as

$$J_\xi\frac{d\omega_\xi}{dt}+\omega_\eta\omega_\zeta\left(I_\zeta-I_\eta\right)=M_\xi-\sum_{i=1}^{n}L_{\xi i};$$
$$J_\eta\frac{d\omega_\eta}{dt}+\omega_\zeta\omega_\xi\left(I_\xi-I_\zeta\right)=M_\eta-\sum_{i=1}^{n}L_{\eta i};\tag{3}$$
$$J_\zeta\frac{d\omega_\zeta}{dt}+\omega_\xi\omega_\eta\left(I_\eta-I_\xi\right)=M_\zeta-\sum_{i=1}^{n}L_{\zeta i},$$

where J_ξ, J_η, J_ζ – aggregate principal moments of inertia relative to axes *C*ξ, *C*η, *C*ζ; ω_ξ, ω_η, ω_ζ – projection of the angular velocity of the body on the axis *C*ξ, *C*η, *C*ζ; M_ξ, M_η, M_ζ – points of pressures relative connected with the aggregate of axes *C*ξ, *C*η, *C*ζ; $L_{\xi i}$, $L_{\eta i}$, $L_{\zeta i}$ (i = 1, 2, …, n) – reaction times of elastic supports relatively to the relevant axes.

The projections of the vector of angular velocity of the body on the invariably connected with it axes determine by the formula Euler

$$\omega_\xi=\dot\psi\sin\theta\sin\varphi+\dot\theta\cos\varphi;$$
$$\omega_\eta=\dot\psi\sin\theta\cos\varphi-\dot\theta\sin\varphi;\quad \omega_\zeta=\dot\psi\cos\theta+\dot\varphi.\tag{4}$$

with the notation

$$\frac{d\psi}{dt}=\omega_\psi;\quad \frac{d\theta}{dt}=\omega_\theta;\quad \frac{d\varphi}{dt}=\omega_\varphi\tag{5}$$

write the relation (4) in matrix form

$$\Omega_\xi=G\Omega_\psi,\tag{6}$$

where Ω_ξ i Ω_ψ – matrix-column

$$\Omega_\xi=\mathrm{col}\left(\omega_\xi,\omega_\eta,\omega_\zeta\right),$$
$$\Omega_\psi=\mathrm{col}\left(\omega_\psi,\omega_\theta,\omega_\phi\right);\tag{7}$$

G – square matrix of coordinate transformation

$$G=\begin{pmatrix}\sin\theta\sin\varphi & \cos\varphi & 0\\ \sin\theta\cos\varphi & -\sin\varphi & 0\\ \cos\theta & 0 & 1\end{pmatrix}.\tag{8}$$

Differentiating (6) with time, using (8) we obtain:

$$\frac{d\Omega_\xi}{dt} = G\frac{d\Omega_\psi}{dt} + H\Omega_\psi, \tag{9}$$

$$H = \frac{dG}{dt} =$$

$$= \begin{pmatrix} \omega_\theta \cos\theta \sin\phi + \omega_\phi \sin\theta\cos\phi & -\omega_\phi \sin\phi & 0 \\ \omega_\theta \cos\theta\cos\phi - \omega_\phi \sin\theta\sin\phi & 0 & 0 \\ -\omega_\theta \sin\theta & 0 & 0 \end{pmatrix}. \tag{10}$$

We assume that the load due to dynamic unbalance of the motor mechanisms are defined in the moving sysyem of coordinates $C\xi\eta\zeta$, and reaction of supports - in the fixed $Ox_0y_0z_0$. Projections of the main vector of loads on a fixed axis is found as

$$F_0 = D\cdot F_{\tilde{n}}, \tag{11}$$

where

$$F_0 = \mathrm{col}(F_x, F_y, F_z); \quad F_c = \mathrm{col}(F_\xi, F_\eta, F_\zeta),$$

elements of the matrix D respectively equal

$$d_{11} = \cos\psi\cos\phi - \sin\psi\cos\theta\sin\phi;$$
$$d_{12} = -\cos\psi\sin\phi - \sin\psi\cos\theta\cos\phi;$$
$$d_{13} = \sin\psi\sin\theta;$$
$$d_{21} = \sin\psi\cos\phi + \cos\psi\cos\theta\sin\phi;$$
$$d_{22} = -\sin\psi\sin\phi + \cos\psi\cos\theta\cos\phi;$$
$$d_{23} = -\cos\psi\sin\theta;$$
$$d_{31} = \sin\theta\sin\phi;\ d_{32} = \sin\theta\cos\phi;\ d_{33} = \cos\theta. \tag{12}$$

Here F_ξ, F_η, F_ζ – the main vector projection of loads on the axis of the system $C\xi\eta\zeta$.

Projections of the main vector of reactions of supports on fixed axes found in the form of

$$R_0 = \sum_{i=1}^{n}\left[C_i(X_i - \Xi_i) + N_iV_i\right], \tag{13}$$

where R_0 – matrix-column of the projections of the main vector of reactions,

$$R_0 = \mathrm{col}\left(\sum_{i=1}^{n} R_{xi}, \sum_{i=1}^{n} R_{yi}, \sum_{i=1}^{n} R_{zi}\right); \tag{14}$$

X_i i Ξ_i – matrix-column of the coordinates of control points of aggregate in a fixed invariably linked with the body coordinate systems

$$X_i = \mathrm{col}(x_i, y_i, z_i), \quad \Xi_i = \mathrm{col}(\xi_i, \eta_i, \zeta_i); \tag{15}$$

V_i – matrix-column of the projections of velocity vectors of control points on the fixed axes

$$V_i = \mathrm{col}(v_{xi}, v_{yi}, v_{zi}), \tag{16}$$

C_i i N_i – square matrix stiffness and viscous friction coefficient of elastic support of the power aggregate,

$$C_i = \begin{pmatrix} c_{xi} & c_{xyi} & c_{xzi} \\ c_{yxi} & c_{yi} & c_{yzi} \\ c_{zxi} & c_{zyi} & c_{zi} \end{pmatrix}; \quad N_i = \begin{pmatrix} \upsilon_{xi} & \upsilon_{xyi} & \upsilon_{xzi} \\ \upsilon_{yxi} & \upsilon_{yi} & \upsilon_{yzi} \\ \upsilon_{zxi} & \upsilon_{zyi} & \upsilon_{zi} \end{pmatrix}. \tag{17}$$

The coordinates of the points of securing the power aggregate in a moving coordinate system $C\xi\eta\zeta$ are given geometric characteristics of the aggregate. In the fixed coordinate system $Ox_0y_0z_0$ the coordinates of these points are found in the form

$$X_i = X_{\tilde{n}} + D\cdot\Xi_i, \tag{18}$$

Moreover, the matrix D is defined by relation (12). The velocities of elastic fastening points of the power aggregate in a coordinate system $C\xi\eta\zeta$ found in a

$$V_{ci} = \Lambda_i\Omega_\xi, \tag{19}$$

where V_{ci} – matrix-column of the projection of velocities,

$$V_{ci} = \mathrm{col}(v_{\xi i}, v_{\eta i}, v_{\zeta i});$$

Λ_i – square matrix,

$$\Lambda_i = \begin{pmatrix} 0 & \zeta_з & -\eta_з \\ -\zeta_з & 0 & \xi_з \\ \eta_з & -\xi_з & 0 \end{pmatrix}. \tag{20}$$

The connection of the projections of velocity of fixing points of the body in fixed $Ox_0y_0z_0$ and moving $C\xi\eta\zeta$ coordinate systems is expressed with dependence

$$V_i = DV_{ci}\,. \tag{21}$$

Considering dependences (6), (19), (21) we obtain

$$V_i = D\Lambda_i G\Omega_\psi\,. \tag{22}$$

Substituting in equation (13) expressions (18) and (22), we obtain

$$R_0 = \sum_{i=1}^{n}\left(C_i\left[X_c + (\mathrm{D}\text{-}1)\Xi_i\right] + N_i\mathrm{D}\Lambda_i\mathrm{G}\Omega_\psi\right). \tag{23}$$

Considering (11), (23), equations of translational motion of the center of mass of the power aggregate (1), (2) reduce to the form

$$m\frac{dV_c}{dt} = F_0 - R_0; \quad \frac{dX_c}{dt} = V_c, \qquad (24)$$

where

$$X_c = \mathrm{col}(x_c, y_c, z_c); \quad V_c = \mathrm{col}(v_{cx}, v_{cy}, v_{cz}).$$

The equation of a spherical movement of the power aggregate (3) is rewritten using (6) and (9) in matrix form

$$J \cdot \left(G\frac{d\Omega_\psi}{dt} + H\Omega_\psi \right) + K\Omega = M - L, \qquad (25)$$

where J – diagonal matrix of the main central moments of inertia of the aggregate,

$$J = \mathrm{diag}\left(J_\xi, J_\eta, J_\zeta\right);$$

K – square matrix

$$K = \begin{pmatrix} J_\xi - J_\eta & 0 & 0 \\ 0 & J_\xi - J_\zeta & 0 \\ 0 & 0 & J_\eta - J_\xi \end{pmatrix};$$

Ω – matrix-column,

$$\Omega = \begin{pmatrix} \left(\omega_\psi \sin\theta\cos\phi - \omega_\theta \sin\phi\right)\left(\omega_\phi - \omega_\psi \cos\theta\right) \\ \left(\omega_\phi - \omega_\psi \cos\theta\right)\left(\omega_\psi \sin\theta\sin\phi - \omega_\theta \cos\phi\right) \\ \left(\omega_\psi \sin\theta\sin\phi - \omega_\theta \cos\phi\right)\left(\omega_\psi \sin\theta\cos\phi - \omega_\theta \sin\phi\right) \end{pmatrix};$$

M i L – matrix-columns of the projections of the main moments of loads and the main moments of the reactions of supports on axes invariably associated with the power aggregate,

$$M = \mathrm{col}(M_\xi, M_\eta, M_\zeta); \quad L = \mathrm{col}\left(\sum_{i=1}^{n} L_\xi, \sum_{i=1}^{n} L_\eta, \sum_{i=1}^{n} L_\zeta\right).$$

Projections of the main moment of loads are determined in a coordinate system $C\xi\eta\zeta$ based on the analysis of the work of mechanisms of the aggregate. Projections of the main moment of the reactions of supports found with (23) in the form

$$L = \sum_{i=1}^{n} \Lambda_i D1\left\{C_i\left[X_c + (D-1)\Xi_i\right] + N_i D\Lambda_i G\Omega_\psi\right\}, \qquad (26)$$

where Λ – square matrix what is determined by the dependence (20), $D1 = D^{-1}$, moreover

$$d1_{11} = \cos\psi\cos\phi - \sin\psi\cos\theta\sin\phi;$$
$$d1_{12} = \sin\psi\cos\phi + \cos\psi\cos\theta\sin\phi;$$
$$d1_{13} = \sin\theta\sin\phi;$$
$$d1_{21} = -\cos\psi\sin\phi - \sin\psi\cos\theta\cos\phi;$$
$$d1_{22} = -\sin\psi\sin\phi + \cos\psi\cos\theta\cos\phi;$$
$$d1_{23} = \sin\theta\cos\phi;$$
$$d1_{31} = \sin\psi\sin\theta; \quad d1_{32} = -\cos\psi\sin\theta;$$
$$d1_{33} = \cos\theta.$$

Thus, taking into account the dependence (5), (7), (25), (26), the equation of motion of a spherical power aggregate write in the form

$$J\left(G\frac{d\Omega_\psi}{dt} + H\Omega_\psi\right) + K\Omega =$$
$$= M - \sum_{i=1}^{n} \Lambda_i D^{-1}\left\{C_i\left[X_c + (D-1)\Xi_i\right] + N_i D\Lambda_i G\Omega_\psi\right\};$$
$$\frac{d\Psi}{dt} = \Omega_\psi, \qquad (27)$$

moreover,

$$\Psi = \mathrm{col}\left(\psi, \theta, \phi\right).$$

Based on the proposed mathematical model the dynamics of the power aggregate YaMZ 238 with the following suspension options has been explored: three-support, four-support, five-support are considered in our research.

It was found that the amplitude of oscillations depend strongly on both the crankshaft speed of power aggregate and the parameters of its pendant. With increasing of cyclic frequency in the range of 80 rad/s to 220 rad/s the amplitudes are monotonically decreasing. The largest were vertical displacement amplitudes of the center of mass of the aggregate and the smallest were the amplitudes of horizontal displacement in the direction of the axis *Oy*. Comparing the maximum significance of the amplitudes of translational displacements of the center of mass of the power aggregate obtained for different types of suspension, it can be concluded that the most appropriate is three-support suspension of the power aggregate. Three-support and five-support suspensions are characterized by large amplitudes of oscillations of the center of mass of the aggregate, which can be explained by the narrowing of the resonance between the field of oscillations of mechanical system due to increase of its rigidity.

4 Conclusion

A mathematical model of the spatial vibration powertrain bus enables to conduct a comprehensive analysis of the impact of the structure and elastic-dissipative characteristics of suspension elements on the parameters of vibration of the unit. The model provides the necessary precision vibration analysis process, since it takes into account the geometric nonlinearity of the mechanical system and gyroscopic effects that inevitably occur during spherical rigid body motion. It is shown that reducing the amplitude of vibration of powertrains cannot always be achieved by increasing the stiffness of the suspension. To achieve this goal it is necessary to implement operational modes implemented in the frequency domain interresonant mechanical system.

References

[1] Ломакин В. В. *Расчет крутильных колебаний в трансмиссии полноприводного легкового автомобиля при движении по неровным дорогам и оптимизация параметров демпфирующей муфты* / В. В. Ломакин, Нгуен Гуй Чыонг // Машиностроение «Известия ВУЗов». – 2008. – №1. – С. 50–56.

[2] Ломакин В. В. *Расчет колебаний силового агрегата автомобиля путем оптимизации параметров его опор* / В. В. Ломакин, Нгуен Гуй Чыонг // Известия МГТУ «МАМИ». – 2008. – №1(5). – С. 72–79.

[3] Горбаха М. М. *До питання переобладнання вантажних автомобілів в Україні* / М. М. Горбаха // Системні методи керування, технологія та організація виробництва, ремонту і експлуатації автомобілів. – 2002. – С. 65–68.

[4] Альдайуб Зияд. *К вопросу о поиске оптимальных решений для рамы грузового автомобиля на базе уточненных конечно-элементных модедей* / Альдайуб Зияд, В. Н. Зузов // Известия ВУЗов. Машиностроение. – 2005. – № 12. – С. 46–66.

[5] Ali El Hafidi. *Vibration reduction on city buses: Determination of optimal position of engine mounts* / Ali El Hafidi, Bruno Martin, Alexandre Loredo, EricJego // Mechanical Systems and Signal Processing. – 2010. – №24. – С. 2198–2209.

[6] Claes Olsson. *Active automotive engine vibration isolation using feedback control* / Claes Olsson // Journal of Sound and Vibration. – 2006. – №294. – С. 162–176.

[7] Тольский В. Е. *Колебания силового агрегата автомобиля* / В. Е. Тольский, Л. В. Корчемный, Г. В. Латышев, Л. М. Минкин. – М. : Машиностроение, 1976. – 266 с.

[8] Ганиев Р.Ф. *Колебания твердых тел* / Р. Ф. Ганиев, В.О. Кононенко. – М. : Гл. ред. физ.-мат. литературы. Изд. «Наука», 1976. – 432 с.

[9] Смерека І. П. *Дослідження геометрії мас силового агрегата колісного транспортного засобу* / І. П. Смерека, В. М. Палюх // Проектування виробництво, та експлуатація автотранспортних засобів та поїздів. – Вип. 8. – 2004. – С. 96–106.